PB-251 178

ESTIMATION OF THE COST OF CAPITAL FOR MAJOR UNITED STATES INDUSTRIES WITH APPLICATION TO POLLUTION CONTROL INVESTMENTS

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Prepared for:

Environmental Protection Agency

November 1975

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NTIS

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U. S. DEPARTMENT OF COMMERCE

BIBLIOGRAPHIC DATA SHEET	1. Report No. EPA-230/3-76-001	2.	3. Recipient's Accession No.

4. Title and Subtitle
Estimation of the Cost of Capital for Major U.S. Industries With Application to Pollution Control Investments

5. Report Date
Nov 1975

6.

7. Author(s)
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8. Performing Organization Rept. No.

9. Performing Organization Name and Address
Dr. Gerald A. Pogue
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10. Project/Task/Work Unit No.

11. Contract/Grant No.
Contract # 68-01-2848

12. Sponsoring Organization Name and Address
Environmental Protection Agency
Office of Planning & Evaluation
401 M St., S.W., Washington, D.C. 20460

13. Type of Report & Period Covered

14.

15. Supplementary Notes

16. Abstracts

This study estimates the historical and current costs of equity and total capital faced by firms by six major U.S. industries. In estimating equity costs, traditional DCF and the capital asset pricing model approaches have been used.

The estimates are extrapolated into the future, under the assumption that future values will reflect an average of past risk premiums, and a methodology specified that can be used to measure the financial impact on firms of investments in pollution control equipment.

17. Key Words and Document Analysis. 17a. Descriptors

Cost of Capital, Business risk, financial risk, pollution control investment, equity, debt

17b. Identifiers/Open-Ended Terms

17c. COSATI Field/Group

PRICES SUBJECT TO CHANGE

18. Availability Statement Unlimited	19. Security Class (This Report) UNCLASSIFIED	21. No. of Pages 255
	20. Security Class (This Page UNCLASSIFIED	22. Price 9.00-2.25

093131

PB 251 178

EPA-230/3-76-001
November 1975

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Contract No. 68-01-2348

Environmental Protection Agency
Office of Planning and Evaluation
Economic Analysis Division

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SPRINGFIELD, VA. 22161

ESTIMATION OF THE COST OF CAPITAL FOR MAJOR UNITED STATES INDUSTRIES WITH APPLICATION TO POLLUTION-CONTROL INVESTMENTS

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November 1975

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PREFACE

In this report I have attempted to draw together a number of elements of capital markets and corporate finance theories in order to derive new procedures for estimating the costs of capital for major United States industries. The report presents no new theory; instead, the major task has been to combine and apply existing theories to practical estimation and decision problems.

The major part of the report is concerned with the cost of capital estimation methodology and empirical results. In addition, I have begun the development of an analytical framework for analyzing the economic impact on corporations of mandated investment programs in pollution-control devices. My approach combines the cost of capital estimates with capital budgeting procedures to produce Financial Impact Indices of the effect on corporations of pollution control requirements. However, no attempt has been made to present a detailed analysis of this question, but rather to point the direction in which the analysis might fruitfully proceed.

The report deals with many concepts from finance theory, some of which are relatively new. I have tried to make the report reasonably self contained by including two lengthy parts dealing with the relevant theoretical background; Part 2 deals with the cost of capital theory, Part 3 with modern portfolio and capital markets theories. However, even these discussions assume some prior knowledge of financial management concepts. It is assumed that the reader has had a course in financial management of about the level of the James C. Van Horne text (*Financial Management and Policy*, Second Edition, Prentice Hall, 1971) or the J. Fred Weston and Eugene Brigham text (*Managerial Finance*, Fourth Edition, Holt Rinehart & Winston, 1972). For those without this background, particularly diligent reading of Parts 2 and 3 will be necessary. In addition, since the cost of capital estimation procedures use econometric models, some knowledge of basic econometrics and statistics would be helpful. A particularly good source is the J. Johnston text (*Econometric Methods*, Second Edition, McGraw-Hill, 1972).

In the conduct of the research and the writing of this report, I have relied heavily on the writings of my former colleague, Professor Stewart C. Myers of MIT. Professor Myers has done much pathbreaking research in the corporate-finance area. I have relied particularly on his writings in the cost-of-capital and capital-budgeting areas. This research has also benefited from my private conversations with Professor Robert Litzenberger of Stanford University.

Finally, I would like to express my appreciation to my research associate Mai Pogue, to my research assistants C. Nguyen and M. Castellino, and to Anna Beliveau who has diligently typed the drafts of this report and provided valuable editorial assistance.

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November 1975

ESTIMATION OF THE COST OF CAPITAL FOR MAJOR UNITED STATES INDUSTRIES WITH APPLICATION TO POLLUTION-CONTROL INVESTMENTS

PART 1. INTRODUCTION

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November 1975

I. INTRODUCTION

(a) Background and Purpose of Study

The cost of capital has long been recognized as an important ingredient in financial decision-making. In capital budgeting, for example, it permits the condensation of a stream of future cash flows into a single figure of merit for an investment project or strategy—its discounted present value. The cost-of-capital concept is prominent in both the literature and practice of finance: textbook definitions abound; the term is common in management conversations.

The application of the concept in practice, however, is not at all simple or straightforward. While much agreement exists regarding the general concept, less exists as to how the cost of capital should be measured and applied in practice. While part of the difficulty lies in the abstraction of the usual definitions, the major problem stems from the lack of simple and reliable estimation procedures.

Most of the past academic studies of cost of capital have focussed on public utilities. This was due in part to the regulated nature of the utility industry (and thus the public interest and availability of data) and in part to the risk structure of the industry. As most public utilities face reasonably similar investment risks, the industry could be approximately considered as made up of companies of similar risk, thus sidestepping the difficult question of how the cost of capital varied with investment risk.

The purpose of this study is to estimate capital costs for a broad cross-section of American industries. As such, it is one of the first empirical studies to explicitly consider differences in investment risk. To achieve this end, the study combines traditional security valuation approaches to estimation of capital costs with modern capital markets theory to develop new estimation procedures which reflect the current state of finance theory.

While the procedures used in this study represent improvements over previous studies, they must not be considered as either infallible or the final word on the subject. Finance theory is evolving rapidly and new methods for estimating capital costs will undoubtedly appear. Nevertheless, while the final answer to the estimation problem is not available (and will not be for some time), a great deal is known upon which to base useful estimates of capital costs.

In conjunction with micro-economic analyses being performed for and by the Environmental Protection Agency, cost-of-capital estimates are produced for six basic industry groups: pulp and paper, chemicals, petroleum refining, iron and steel, non-ferrous metals, and utilities. Estimates were made as of the year-ends 1971 through 1974 and projections prepared for each year of the 1975-1984 period. The estimates include the cost of equity capital and the weighted average cost of capital.

II. ORGANIZATION OF THE REPORT

The report is divided into five parts. Part 2 presents the relevant cost-of-capital concepts which underlie the estimation procedures. Section I contains a definition of the cost of equity capital. Section II describes three related discounted cash flow security valuation models which are used to estimate the cost of equity capital. Section III deals with the impact of debt financing on the market value of the firm and procedures for measuring the weighted average cost of capital.

Part 3 summarizes the necessary capital markets and portfolio theory concepts. Sections II and III discuss how the risk of common stocks can be measured, and how risk is related to the cost of capital. Section IV summarizes the empirical literature on the relationship between stock-market-oriented risk measures and the accounting characteristics of firms' assets.

Part 4 presents the cost-of-capital estimation methodology and empirical results. Section II shows how the discounted cash flow cost of capital models and the risk-return concepts of Parts 2 and 3, respectively, can be combined to produce cross-sectional estimation equations. Section III describes the primary and secondary company samples used in the study. Section IV presents the equity risk measures for sample firms. Section V describes the procedures used to estimate the cross-sectional regression variables and presents the cost of equity capital estimates. Section VI contains the methodology used to estimate weighted average costs of capital and presents the results. Finally, Section VII shows how the weighted costs are likely to change during the 1975-84 period given estimates of future interest rates.

The purpose of Part 5 is to show how the cost-of-capital estimates developed in Part 4 can be used for making capital budgeting decisions in practical situations. Section II reviews and extends the capital budgeting procedures introduced in Part 2. Section III begins the development of analytical procedures which can be used for measuring the financial impact on corporations of investments in pollution-control devices.

ESTIMATION OF THE COST OF CAPITAL FOR MAJOR UNITED STATES INDUSTRIES WITH APPLICATION TO POLLUTION-CONTROL INVESTMENTS

PART 2. THEORY: COST OF CAPITAL CONCEPTS

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November 1975

I. INTRODUCTION

The purpose of Part 2 is to present the relevant cost-of-capital concepts underlying the models and estimation procedures of Part 4.

Section II contains definitions of the cost of capital. For ease of exposition it is assumed in Section II that the firms under consideration are all-equity financed. The definition is broadened later to include debt financing (See Section IV).

In Section III three related "discounted cash flow" security valuation models are discussed. The models provide the basic framework for estimating the firm's cost of equity capital.

Section IV deals with the impact of debt financing on the market value of the firm and generalizes the cost-of-capital definitions and Net Present Value (NPV) decision rules to include the effects of debt financing.

II. THE COST OF CAPITAL FOR AN "ALL-EQUITY" FIRM

(a) Definition[1]

The cost of capital for an all-equity-financed company is the rate of return that investors expect to earn on the company's securities, as well as on all other securities of equivalent risk. The expected rates of return on securities of equivalent risk must be equal. If one security offers a higher rate of return than other equivalent risk securities, then there will be excess demand for this security and excess supplies of the others. Conversely, a security offering a relatively low return will be in excess supply. Prices rise when there is an excess demand and fall when there is an excess supply. Thus prices of equivalent risk securities will tend to adjust so that they will offer the same expected rate of return.

The cost of capital is the minimum acceptable rate of return or "hurdle rate" for new investments by the firm. The logic in developing a cost of capital on an all-equity-financed firm's investment goes as follows:

1 The firm is one of a class with similar risk characteristics—call this class "j".

2. At any point in time there is a unique expected rate of return prevailing in capital markets for this degree of risk—call it R_j.
3. The share price of the firm in question will adjust so that it offers an expected rate of return R_j to investors.
4. This rate, the shareholders' opportunity cost, should be the minimum acceptable expected rate of return on new investment, assuming the projects under consideration have risk characteristics similar to currently held assets.

(Suppose the firm undertakes investments offering less than R_j; its shareholders will be worse off than if the firm had foregone the investment and paid an extra dividend, since the shareholders can always invest at R_j. On the other hand, investors can obtain no more than R_j by investing directly, so the firm should never pass up investments offering more than R_j.)

For any stock in risk class j, the cost of equity capital is formally defined by the relation

$$P_{tj} = \frac{D_{t+1,j} + P_{t+1,j}}{1 + R_j} \qquad (2\text{-}1)$$

where

P_{tj} = ex-dividend price of the stock at the end of period t

$D_{t+1,j}$ = dividends expected during period t+1 [2]

$P_{t+1,j}$ = expected ex-dividend price for t+1

Of course, it is not literally true that everyone has the same expectations of future return, but for purposes of analysis this is assumed so that it is permissible to speak of "the market's" expectations.

The above logic must be extended when the project under consideration has risk characteristics different from currently held assets. Suppose the project under consideration has risk characteristics more like firms in risk class k. For example, assume a paper company is considering expanding into the chemical business. In this case, the stockholder's opportunity cost for this investment is R_k, the going rate of return of firms in risk class k. Thus, the rate R_k should be used to discount projects of this type.

For investments in risk class k, the cost of equity capital can be formally defined by extending Equation (2-1).

$$\Delta P_t = \frac{\Delta D_{t+1} + \Delta P_{t+1}}{1 + R_k} \qquad (2-2)$$

where

ΔP_t = the change in the price at time t due to the adoption of the project in risk class k

ΔD_t = the expected incremental cash flow from adopting the project

ΔP_{t+1} = change in price at time t+1 due to the adoption of the project

The cost of capital, as defined above, is relatively straightforward. The underlying proposition is that at any point in time securities are so priced that all securities of equivalent risk (i.e., all securities in a risk class) offer the same expected rate of return. For a given stock, however, the basic problem is to determine the expected rate of return for the class in which the stock falls, that is, estimating the rates prevailing in the market. There is no simple way of doing this; measurement of expectations is intrinsically difficult. The security valuation theory which permits estimation of stockholders' required returns is presented in Section III.

(b) Decision Rules

The premise in finance theory is that corporate managers will make decisions in order to increase the wealth of the owners of the firm. In other words, the corporation is assumed to be run primarily for the long-run benefit of the common stockholders. The decision rules proposed by the theory are designed to help achieve that end. The cost-of-capital concept is no exception.

Consider first a firm which is all-equity financed. The cost of capital for such a firm is the rate of return that investors expect to earn on the company's common stock, as well as all other securities of equivalent risk. (The expected rate of return on all securities of equivalent risk must be equal.) This rate is the stockholders' opportunity cost, since they can always expect to earn this rate by investing in other firms of comparable risk.

Thus, the cost of capital is the minimum acceptable rate of return, or "hurdle rate", for new investments by the firm. If the firm were to undertake an investment with rate of return less than the cost of capital, its shareholders would be worse off than if the firm had foregone the investment and paid an extra dividend, since the stockholders can always invest elsewhere and earn the cost of capital. On the other hand, investors can expect to earn no more than the cost of capital by investing elsewhere (at the same risk level), so the firm should never pass up investments offering more than the cost of capital.

Thus, a decision rule to increase stockholder wealth is to accept all projects which offer an internal rate of return (IRR) greater than the cost of equity capital (R). That is

$$\text{Accept Project if } IRR > R \tag{2-3}$$

Another version of this rule is to accept all projects for which the sum of all future cash flows discounted at the cost of capital exceeds the investment. The difference between discounted benefits and costs is called the net present value (NPV) of the project

$$NPV = \sum_{t=1}^{T} \frac{C_t}{(1+R)^t} - I_0 \qquad (2\text{-}4)$$

where

C_t = the expected project cash flow in year t

I_0 = the investment (assumed to be made at $t = 0$)

T = the economic lifetime of the project

R = the cost of (equity) capital

$$\text{Accept Project if } NPV > 0 \qquad (2\text{-}5)$$

When the firm has both debt and equity financing, the goal is the same. Accept only projects which will increase stockholder wealth. The NPV decision rule will be generalized in Section IV to include debt financing.

III. SECURITY VALUATION THEORY FOR ESTIMATING THE COST OF EQUITY CAPITAL

The usual approach to estimating the cost of equity capital is through the use of various security valuation models. The valuation models relate the cost of equity capital to the expected future dividends on the firm's outstanding stock. The rate of return implicit in this stream of dividends is that discount rate which equates the present value of the dividends to the current share price.

The general form of the valuation model will be considered first, followed by the three specific forms used in this study.

General Form

The basic discounted-cash-flow model for security valuation follows directly from Equation (2-1). The current price P_0 (the subscript j is now unnecessary) is given by

$$P_0 = \frac{D_1 + P_1}{(1 + R)}$$

it similarly follows from Equation (2-1) that P_1 can be replace by $(D_2 + P_2)/(1 + R)$. Therefore,

$$P_0 = \frac{D_1}{(1+R)} + \frac{D_2}{(1 + R)^2} + \frac{P_2}{(1 + R)^2}$$

If the substitution is continued for P_2, P_3, etc., we obtain

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + R)^t} \qquad (2\text{-}6)$$

Equation (2-6) states that the current common stock share price is the present value of the stream of future expected dividends, where the discount rate (the cost of equity capital) reflects both the time value of money and the riskiness of the stream.

The discounted-cash-flow method is useful because it is often possible to arrive at reasonable estimates of the firm's dividend stream. Since P_0 is known, estimates of D_1 D_2 . . . yields an estimate of R. Three simplifications of Equation (2-6) which permit reasonably straightforward estimation of future dividends will now be considered.

Perpetual-Growth Model

Suppose the dividend stream D_t is expected to grow indefinitely at some rate g which is less than R. Then Equation (2-6) can be simplified to

$$P_0 = \frac{D_1}{R - g} \qquad (2\text{-}7)$$

Therefore,

$$R = \frac{D_1}{P_0} + g \qquad (2\text{-}8)$$

That is, the cost of capital will be equal to the dividend yield plus the growth rate if the stated assumption is correct. (See Appendix A of Part 2 for the development of Equation (2-7).)

For companies for which a constant long-term trend in earnings and dividends is realistic (for example, utilities), Equation (2-8) can be a reasonable rule of thumb for estimating R.

For some companies, however, if Equation (2-8) is applied in a mechanical way, the estimates of R will be upward biased. Companies in their early stages typically have a period of rapid growth of earnings and dividends followed by a much slower growth rate as they enter maturity (for example, IBM). During this period of rapid growth, if estimates of future growth rates are mechanically produced from past trends, the estimated g will overstate the market's expectations. The result will be an upward-biased estimate of R.

It is often stated that estimates produced from Equation (2-8) based on mechanical extrapolations of past trends tend to overstate the cost of capital. This view is based on the assumption that future growth rates will tend to be less than or equal to historical rates; non-mature companies will mature (i.e., their real growth opportunities will diminish) and mature companies will continue or expire. If this view is correct, then estimates of R from Equation (2-8) can be considered as maximum values for the cost of equity capital.

No-"Real"-Growth Model

Consider next the case where all future earnings not paid out as dividends are reinvested in projects which offer an expected return exactly equal to the cost of equity capital.

In this case, the general-valuation model, Equation (2-6), reduces to

$$P_0 = \frac{E_1}{R} \tag{2-9}$$

Therefore

$$R = \frac{E_1}{P_0} \tag{2-10}$$

The cost of equity capital is equal to the expected earnings per share during the next year (E_1) divided by the current share price (P_0). (See Appendix A for the development of Equation (2-9).)

The expression "no real growth" needs further explanation. Real growth refers to reinvestment at rates of return different from the cost of capital. If the firm reinvests future earnings in projects yielding exactly the cost of capital, nothing will change. The investors will simply have given up current dividends at the time of the reinvestment for a future stream which has exactly the same present value. Thus, the adoption of these projects will make the firm no more or less attractive to investors. The firm's stock price will not change, nor will its earnings per share (E_1). Therefore, the earnings-price ratio will still equal the cost of equity capital.

If the project offers a future rate of return in excess of the cost of equity capital, the stock price will rise, current current earnings per share will remain constant, and the earnings-price ratio will understate the cost of equity capital.

For example, consider a firm initially earning and paying out $10 per share per year. This dividend is not expected to grow or decline over time. R, the cost of equity capital, is 10%, so stock price is $100. Now the firm announces that it will invest the first year's earnings in a project expected to yield x dollars per year in perpetuity, starting in t = 2. The expected rate of return on this project is x/10.

Table (2-1) shows, for different levels of x, the errors in using the earnings-price ratio as a measure of R. Note that the measure is correct only in the case in which the additional investment offers a 10% return.

Table 2-1

THE RELATIONSHIP BETWEEN THE EARNINGS PRICE RATIO AND REINVESTMENT RATES OF RETURN

x (Dollar Return Per Year)	x/10 (Expected Yield on Project)	P_0 (100 + Net Present Worth of Project)	EPSI/P_0 (Earnings-Price Ratio)	R (Cost of Equity Capital)
$ .50	0.05	95.45	0.105	0.10
$1.00	0.10	100.	0.100	0.10
$1.50	0.15	104.55	0.096	0.10
$2.00	0.20	109.09	0.92	0.10

Source: S. C. Myers [18], Table 1, p. 14.

The usual view is that earnings-price ratios tend to understate the cost of equity capital. This is based on the assumption that corporate managers will accept only those projects with expected returns greater than or equal to R.

To the extent that real growth opportunities exist, then, the earning-price ratio will understate the cost of equity capital. In this case, the estimates of R from Equation (2-10) can be considered as minimum values for the cost of equity capital.

Finite-Horizon-Growth Model

The third valuation model makes assumptions about investment behavior which are between the extremes of the previous two. Suppose the firm can reinvest earnings at expected rates or return greater than the cost of equity capital, but only for a finite number of years, T. Beyond T all real growth opportunities cease; any further reinvestment earns only the cost of capital. Thus, the firm will have T years of rapid growth, followed by an indefinite period of normal growth. Given these assumptions, the share price is given by

$$P_0 = \frac{E_1}{R} + I_1 \left[\frac{r - R}{R} \right] T \qquad (2\text{-}11)$$

where

I_1 = the investment per share in period 1 (i.e., retained earnings)

r = the rate of return on reinvested earnings for years 1 through T

$(r \neq R)$

Therefore,

$$R = \frac{E_1}{P_0} + \frac{I_1}{P_0} [r - R]T \qquad (2\text{-}12)$$

As seen from Equation (2-12), the absence of real growth opportunities (i.e., r = R), R is simply the earnings-price ratio. It can also be shown that when the real growth opportunities are expected to last indefinitely, R is again given by $D_1/P_0 + g$. (See Appendix A for the development of Equation (2-11).)

Because of the more flexible and realistic assumptions of this model, there is no *a priori* reason to expect that estimates of R will be either upward or downward biased.

IV. THE WEIGHTED AVERAGE COST OF CAPITAL[3]

(a) The Effects of Leverage on the Market Value of the Firm

Modigliani and Miller (MM) [15] were the first to rigorously consider the question of how the value of a firm changes with financial leverage. Their main conclusion is that in perfect capital markets and no taxes, the value of the firm will be independent of the proportion of debt and equity in the firm's capital structure.

The adjective "perfect" is used to describe markets in which no trader is large enough to have any control over market prices, and in which distorting elements such as transaction costs and taxes can safely be ignored.

The Modigliani and Miller Argument

Suppose that we observe a group of firms which are identical except for financing. We observe the relative value of firms which are all-equity financed (class A) and of the others (class B) which have $4.00 of debt outstanding. The MM proposition implies that

$$V_A = D_B + E_B \qquad (2\text{-}13)$$

e.g.,

$$\$10.00 = \$4.00 + \$6.00$$

where

V_A = the market value of firm A
D_B = the market value of the debt of firm B
E_B = the market value of the equity of firm B

There are markets for unleveraged firms, for the equity of leveraged firms, and for debt.

To "prove" the proposition we consider what happens if the prices are not in the predicted relationship. What if

$$V_A > D_B + E_B \qquad (2\text{-}14)$$

e.g.,

$$\$10.00 > \$4.00 + \$5.00?$$

In this case, who would ever buy shares in the unleveraged firm? Since we assume that all firms under consideration are exactly the same except for financing, an investor who buys up the outstanding debt and equity of a leveraged firm (for $9.00) obtains exactly the same commodity as one who buys up all the shares on an unleveraged firm (for $10.00).

On the other hand, what if

$$V_A < D_B + E_B \qquad (2\text{-}15)$$

e.g.,

$$\$10.00 < \$4.00 + \$7.00?$$

Consider the options open to investors who wish to hold the equity of leveraged firms. Direct purchase costs $7.00, but they also have the option of buying unleveraged firms for $10.00, selling $4.00 of debt, and thereby obtaining a residual claim for a net cost of $6.00. The residual claim is exactly equivalent to the leveraged equity of class B firms. In this case we would expect the value of unleveraged firms to rise and the value of leveraged firms to fall until the MM proposition is satisfied.

In this case, at least, we see that market processes tend to the result that the market value of firms is independent of the proportions of debt and equity financing used. Since advantageous trading opportunities are created at every point where the MM proposition is violated, and since equilibrium in capital markets may be defined as the absence of favorable trading opportunities, the MM proposition is a necessary condition for equilibrium in security markets. (That is, at equilibrium all securities of equivalent risk must have the same expected rates of return, thus eliminating the possibility of making "easy money" through combinations of trades.)

Effects on Equilibrium Expected Rates of Return on the Firm's Stock

Increased financial leverage increases the expected rate of return on the firm's stock, but it also increases the risk borne by stockholders. Table (2-2) presents an example illustrating this. The firm is expected to generate $10.00 per year operating income, but with $7.00 per year standard deviation. Assume that MM are right, and that the firm can borrow at 8 percent. Then the expected rate of return to equity goes up. But so does the risk: as debt increases, the risk of the operating earnings is concentrated on a smaller and smaller equity base.

Modigliani and Miller Propositions I and II

The results of these arguments are the famous MM propositions:

Proposition I. The value of the firm is independent of the proportion of debt and equity in the firm's capital structure. It will always be equal to its value under all-equity financing (V_0).

$$V = D + E = V_0 \qquad (2\text{-}16)$$

Proposition II. The cost of equity capital increases as a function of the leverage ratio, (D/E). The relationship is given by

$$k = \rho_0 + (\rho_0 - i) \cdot \left(\frac{D}{E}\right) \qquad (2\text{-}17)$$

where

k = the cost of equity capital

ρ_0 = the cost of equity capital with all-equity financing (i.e., $D = 0$)

i = the interest rate on debt

Table 2-2

FINANCIAL LEVERAGE—EXAMPLE

The firm has assets of $100 which are expected to produce $10 annually in perpetuity before interest. Assume no taxes. The interest rate is 8 percent.

	% Debt Financing			
	0	25	50	75
Total return	10	10	10	10
Return to debt	0	2	4	6
Return to equity	10	8	6	4
Equity investment	100	75	50	25
Rate of return relative to original equity investment	.10	.107	.12	.16
Standard deviation of equity earnings	7	7	7	7
Standard deviation of return to equity relative to original investment	.07	.093	.14	.28

Source: S. C. Myers [23], Table 2, p. 9.

The reader should note that these proportions are true only if MM's assumptions (perfect capital markets and no taxes) are true. Also assumed is that bankruptcy is costless, that is the firm can be reorganized at no cost to the stockholders.

Corporate Income Taxes and Bankruptcy Costs

Thus far we have ignored a number of things. The most important is the corporate income tax. Since interest payments are deducted from taxable income, increases in leverage reduce taxes and increase the aggregate amount the firm can pay out to investors. Thus, one would expect to increase both the overall value of the firm and the market price of its stock. More precisely, borrowing should increase the market value of the firm by the present value of tax savings due to borrowing.

Tax savings can be extremely valuable. Suppose a firm issues $10.0 million of 6-percent perpetual debt, and that the corporate income tax rate is $T_c = 0.5$. Then the present value of taxes saved[4] is $5.0 million, or one-half the principal amount of the debt issued. The computations are as follows.

The tax "rebate" due to deducting $600,000 from taxable income is $300,000. The present value of $300,000 in perpetuity is

$$\sum_{t=1}^{\infty} \frac{300{,}000}{(1.06)^t} = \frac{300{,}000}{0.06} = \$5{,}000{,}000 \qquad (2\text{-}18)$$

Six percent is the appropriate discount rate because the risk characteristics of the tax rebates are essentially the same as those of the interest payments on the firm's debt.[5] If investors discount the latter at six percent, the same rate should be applied to the former.

Since the present value of tax savings on perpetual debt is simply the tax rate times the amount of debt issues, the MM Proposition I for perpetuities must be modified to:

$$V = D + E = V_0 + T_c D \qquad (2\text{-}19)$$

where

T_c = the corporate income tax rate

V_0 = what the firm would be worth if it were unleveraged

In general, the proposition is:

$$V = D + E = V_0 + \begin{array}{l}\text{present value of tax}\\ \text{savings due to current}\\ \text{and future debt}\end{array} \qquad (2\text{-}20)$$

Example. At the end of 1972, the XYZ Company's book equity was \$565 million. There was no long-term debt. How would the value of the firm change if it issued debt to retire \$170 million of stock? (This would give a 30-percent debt ratio, which is not excessive for an established firm.) According to MM, the firm's value would increase by \$85 million (ΔV).

$$\begin{aligned} \Delta V &= \Delta V_0 + T_c \Delta D \\ &= 0 + 0.5\ (170{,}000{,}000) \\ &= 85{,}000{,}000 \end{aligned}$$

In other words, the tax subsidy to debt financing is a strong incentive indeed. Taken literally, Equation (2-19) implies that the optimal capital structure is 99.99 percent debt.

Of course, this is a lopsided result. It indicates that shareholders always benefit from increased leverage, which is nonsense. A more sensible conclusion follows when we introduce another consideration. This is the apparent existence of "costs of financial distress."

Costs of Financial Distress

At no point has the possibility of financial distress been ignored. We have not assumed that the payment which the holders of the firm's bonds receive will always be the amount promised in the bond contract. What has been assumed is that financial distress is costless, in the special sense that its occurrence in any contingency does not affect the amount which the firm pays out to investors.

However, if the financial distress will reduce the aggregate amount which current investors will receive, then the current value of the firm's securities will be reduced. The amount of the reduction will depend on the probability of "trouble".

The costs associated with financial distress are difficult to pin down precisely, but they can be roughly grouped into three categories.[6]

1. Suppose the firm is threatened with the imminent possibility that available cash flow will be less than its promised interest and/or principal payments. One way to avoid bankruptcy is to change the firm's investment strategy; that is, the firm can pay more to its creditors by investing less. But if valuable investments are thus foregone, then the changes will be costly from the point of view of present investors, even though the cost may be justified if bankruptcy is thereby avoided.
2. Another evasive tactic is to obtain additional financing. In fact, if the firm could always obtain additional financing or revise its capital structure when threatened by financial distress, then there would be no reason for distress actually to occur, and firms could operate at very high debt ratios. However, distress often arrives without many early warning signals, and once the firm is in distress, negotiations between stockholders and creditors are vastly complicated. Both parties find themselves holding high-risk securities, and neither has much incentive to bail the other out.
3. If the firm has enough debt, there will be some contingencies in which bankruptcy cannot be avoided.

Aside from the direct costs, confusion and delay associated with the legal process of reorganization, the firm may be forced to forego investment opportunities which would be feasible and profitable if the firm had avoided bankrupcy by borrowing less. These costs will likewise reduce the present value of the firm.

Practical Implications of the Modigliani-Miller Argument

If the MM argument is accepted in principle, but taxes and costs of distress are taken into account, the market value of the firm will depend on leverage in the way shown by the solid line in Figure 2-1. The optimum is reached when the present value of tax savings due to additional borrowing is just offset by increases in the present value of costs of distress due to additional borrowing. Of course, this assumes that the firm can actually find takers for the relatively high-risk debt that would have to be issued to reach the optimum. If this is not feasible, the MM recommendation becomes "borrow as much as you can." Unfortunately, there is no satisfactory procedure for predicting the debt level at which the optimum would be reached for a given firm.

(b) The Weighted Average Cost of Capital

In the case of all-equity financing, the problem of capital budgeting is reduced to the task of identifying projects with net present values (NPV) greater than zero. The NPV is an estimate of the project's market value (net of

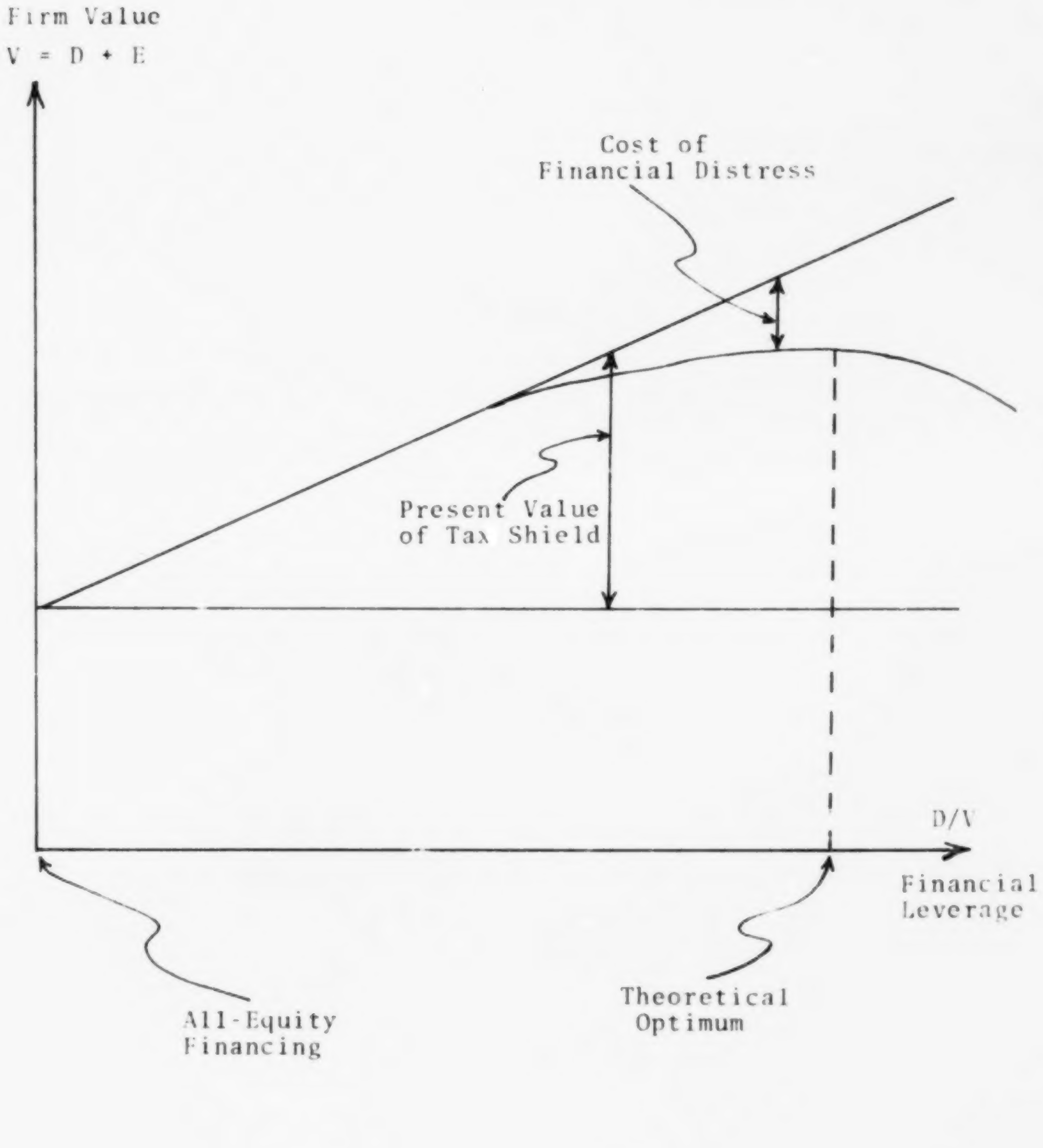
Firm Value
V = D + E
Cost of
Financial Distress
Present Value
of Tax Shield
D/V
Financial
Leverage
All-Equity
Financing
Theoretical
Optimum

Figure 2-1. Impact of leverage on the value of the firm. (Source: S. C. Myers [23], Figure 2, p. 18.)

investment required) if offered to the market as a separately financed enterprise.

When a project is partially financed with debt, the NPV assuring all-equity financing must be adjusted to allow for the positive or negative effects associated with debt financing. The basis for this adjustment is the previously discussed MM valuation formula for the firm (see Equation (2-20)). Assuming a modest amount of debt financing (i.e., the debt ratio lies to the left of the theoretical optimum in Figure 2-1), the value of the firm is equal to the all-equity value plus the present value of the tax savings on current and future debt.

The adjusted present value for a project can similarly be computed from the all-equity NPV plus the present value of the tax savings on debt supported by the project. That is,

$$APV = NPV(\rho_0) + PVTS \tag{2-21}$$

where

APV = the adjusted present value

$NPV(\rho_0)$ = the net present value assuming the project is all-equity financed

PVTS = present value of tax savings on debt supported by the project

The new decision rule is

$$\text{Accept Project if } APV > 0 \tag{2-22}$$

For a project to be acceptable under the base case assumptions of all-equity financing, its internal rate of return (IRR) had to exceed R, the cost of equity capital. It is clear, however, from a comparison of Equations (2-5) and (2-22) that as long as PVTS is greater than zero, the minimum acceptable IRR will now be less than R.

Example. Consider a project which requires an investment of $1000 and produces an expected perpetual return of $90 per year. The cost of capital for all-equity financing is 10 percent (ρ_0).

The base case net present value for the project is given by

$$\text{NPV}(\rho_0) = \sum_{t=1}^{\infty} \frac{90}{(1.1)^t} - 1000$$

$$= -100$$

Thus, assuming all-equity financing, this project should be rejected. The internal rate of return is 9 percent, while the hurdle rate is 10 percent.

Now assume that the project will support $400 of perpetual debt. The interest rate on the debt is 7 percent. The present value of the tax savings is given by

$$\text{PVTS} = \sum_{t=1}^{\infty} \frac{(.07)(400)(.5)}{(1.07)^t}$$

$$= 200$$

The adjusted present value for the project is \$100. Hence it should now be accepted. In the debt financing case, note that the minimum acceptable rate of return on the project is only 8 percent, at which point the APV is exactly equal to zero. This minimum acceptable level of project profitability is called the weighted average cost of capital. It is designated at ρ^*.

Formal Definition of the Weighted Average Cost of Capital, ρ^*

The weighted average cost of capital, ρ^*, is the minimum acceptable rate of return for a project so that acceptance of the project does not reduce the value of the firm's common stock.

The net present value of a project that is partially financed by debt is given by

$$NPV(\rho^*) = \sum_{t=1}^{T} \frac{C_t}{(1+\rho^*)^t} - I_0 \qquad (2\text{-}23)$$

where

C_t = the expected project cash flow in year t

I_0 = the initial investment (assumed to be made at $t = 0$)

T = the estimated lifetime of the project

ρ^* = the weighted average cost of capital

The weighted average cost of capital ρ^* must reflect the financing proportions of the project, the riskiness of the investment and, the relationship between these factors and the economic lifetime of the project.

Now comparison of Equations (2-21) and (2-23) makes it perfectly clear that the adjusted present value and the net present value computed using ρ^* $\left[NPV(\rho^*)\right]$ are measuring exactly the same thing. A project is acceptable if the APV is greater than zero, or correspondingly, if the $NPV(\rho^*) > 0$.

Thus, we can use this relationship between the APV and the $NPV(\rho^*)$ to define ρ^*.

The weighted average cost of capital ρ^* for a given project is the value such that

$$NPV(\rho^*) = APV \qquad (2\text{-}24)$$

or

$$\sum_{t=0}^{T} \frac{C_t}{(1+\rho^*)^t} = \sum_{t=0}^{T} \frac{C_t}{(1+\rho_0)^t} + PVTS \qquad (2\text{-}24a)$$

(Note that in the above equations the investment term, I_0, has been included as the t = 0 term of the summations.)[7]

The minimum acceptable rate of return on a project is obtained by solving Equation (2-24) when the APV is equal to zero.

Example. Consider a project which requires a $1,000 investment and is expected to last for one year only. Assume $\rho_0 = 0.20$. In the base case,

$$NPV(\rho) = \frac{C_1}{1.2} - 1,000$$

where

C_1 = the expected year-end cash flow

Now, assume the project will add \$400 to the firm's debt capacity for one year, and that the borrowing rate is 8 percent. Then the project's total contribution to the firm's value (see Equation (2-21)) is

$$APV = \frac{C_1}{1.2} - 1000 + \frac{.5(.08)(400)}{1.08}$$

$$= \frac{C_1}{1.2} - 1000 + 14.8$$

Setting APV equal to zero and solving for C_1 yields the minimum acceptable C_1 equal to 1182.2. Therefore, the minimum acceptable rate of return on the project, ρ^*, is equal to 0.182 (18.2 percent).

Rules of Thumb for Computing the Weighted Average Cost of Capital

Now we need a simple formula for computing ρ^* as a function of R and other factors. Unfortunately, there is no general-purpose formula. There are some reasonable rules of thumb however.

i. The MM Formula

$$\rho^* = \rho_0(1 - T_c L) \qquad (2\text{-}25)$$

where

ρ_0 = the basic hurdle rate for the firm under all-equity financing

L = the firm's target debt ratio

T_c = the corporate tax rate.

This formula applies only to projects having the same business risk as the firm's existing assets and do not lead to a shift in the firm's target debt ratio.

ii. Generalized MM Formula

$$\rho^*_j = \rho_{0j}(1 - T_c L_j) \qquad (2\text{-}26)$$

where

ρ_{0j} = the hurdle rate for project j under all-equity financing

L_j = the debt capacity ratio for project j (i.e., the proportion of project j that can be debt financed without reducing the firm's other debt capacity).

iii. The Textbook Formula

$$\rho^* = i(1 - T_c)\frac{D}{V} + R\frac{E}{V} \qquad (2\text{-}27)$$

where

i = the current interest rate on the firm's bonds

R = the cost of equity capital

D = market value of debt

E = market value of equity

V = $D + E$

Example. Consider a firm, which is initially all-equity financed, that is expected to earn \$100,000 per year in perpetuity. If $\rho_0 = 0.10$ then

$$V_0 = \frac{100,000}{0.10} = 1,000,000$$

Now it changes its capital structure to include \$500,000 of debt at 7 percent. (The proceeds are used to repurchase \$500,000 of equity.) Following MM we assume the value of the firm, V, increases by the present value of the tax savings generated by the debt

$$V = V_0 + \text{PVTS}$$

$$= 1,000,000 + \frac{17,500}{0.07}$$

$$= 1,250,000$$

The firm now has a (market value) debt ratio of

$$\frac{D}{V} = \frac{500,000}{1,250,000} = 0.40$$

The market value of the equity is 1,250,000 - 500 = 750,000. The expected rate of return on equity is

$$R = \frac{100,000 - .07(500,000) + 0.35(500,000)}{750,000}$$

$$= 0.11$$

We now have the data to compute ρ^* from the textbook formula.

$$\rho^* = i(1 - T_c)\frac{D}{V} + k\frac{E}{V} \qquad (2\text{-}27)$$

$$= .07(.5)(.4) + .11(.6)$$

$$= .08$$

This is the appropriate hurdle rate for a project which offers constant expected cash flows for the indefinite future and which does not change the risk characteristics of the firm's assets or debt ratio of the firm.

We could also have used the MM formula to compute ρ^*.

$$\rho^* = \rho_0(1 - T_c L) \tag{2-25}$$

$$= .10\big(1 - .5(.4)\big)$$

$$= .08$$

Practical Usefulness of the Three Cost-of-Capital Formulas

The various rules for computing the weighted average cost of capital clearly depend on special assumptions. The complete list is given in Table 2-3. The table seems to indicate that the generalized MM rule is superior to the other two, but even its realism must be questioned.

However, we are not really concerned with whether the rules are exactly true, but whether they are useful rules of thumb in a practical context. It has been shown by Myers [21] that the generalized MM rule is reasonably accurate if capital markets are perfect and the rule is properly used. It is beyond the scope of this survey to deal with the question of robustness of the formulas under various violation of the assumptions. This question has been examined in detail by Myers [21] and the interested reader is referred to this source.

In Part 5 the APV formula and its application to investment problems will be further examined.

Table 2-3

NECESSARY AND SUFFICIENT CONDITIONS FOR COST-OF-CAPITAL FORMULAS

Equation	Condition	Formula		
		MM	Generalized MM	Textbook
--	Dividend policy irrelevant	x	x	x
(11a)	Leverage irrelevant except for corporate income taxes	x	x	
(11b)	Investment projects are perpetuities	x	x	x
(11c)	Project does not change firm's risk characteristics	x		x
(11d)	Project makes a permanent contribution to debt capacity	x	x	x
(11e)	Acceptance of project does not lead to shift of target debt ratio	x		x
(11f)	Risk-independence	n.a.*	x	n.a.*
(15a)	Firm's assets expected to generate a constant and perpetual earnings stream			x
(15b)	Firm is already at target debt ratio			x

* n.a. = not applicable.

Source: S. C. Myers, [21], Table 1, p. 14.

(c) Relationship Between ρ^* and Leverage

When the all-equity-financed firm revises its capital structure to include debt, the cost of equity capital (R) will rise because financial leverage makes the firm's common stock even riskier than before.

Figure 2-2 shows how R, i, and ρ^* vary as a function of financial leverage. Because of the tax shield provided by the debt in the capital structure, the weighted average cost of capital will decline with increasing leverage. However, beyond some point (B in Figure 2-2), the possibility of bankruptcy will begin to have increasingly negative consequences. As leverage is further increased, a point will be reached (C in Figure 2-2) beyond which the tax benefits of additional debt are more than offset by the increasing risk of bankruptcy. At this point ρ^* will cease to decline and start to rise.

Generalization of the ρ^* Textbook Formula to Include Other Types of Financing

The formula assumes there are only two kinds of financing instruments. But the weighting principal remains the same even if there are others. Consider the more general case where the firm is financed with debt, preferred and common stock, and financial leases. Since financial leases are simply an alternative form of debt, the first step in computing

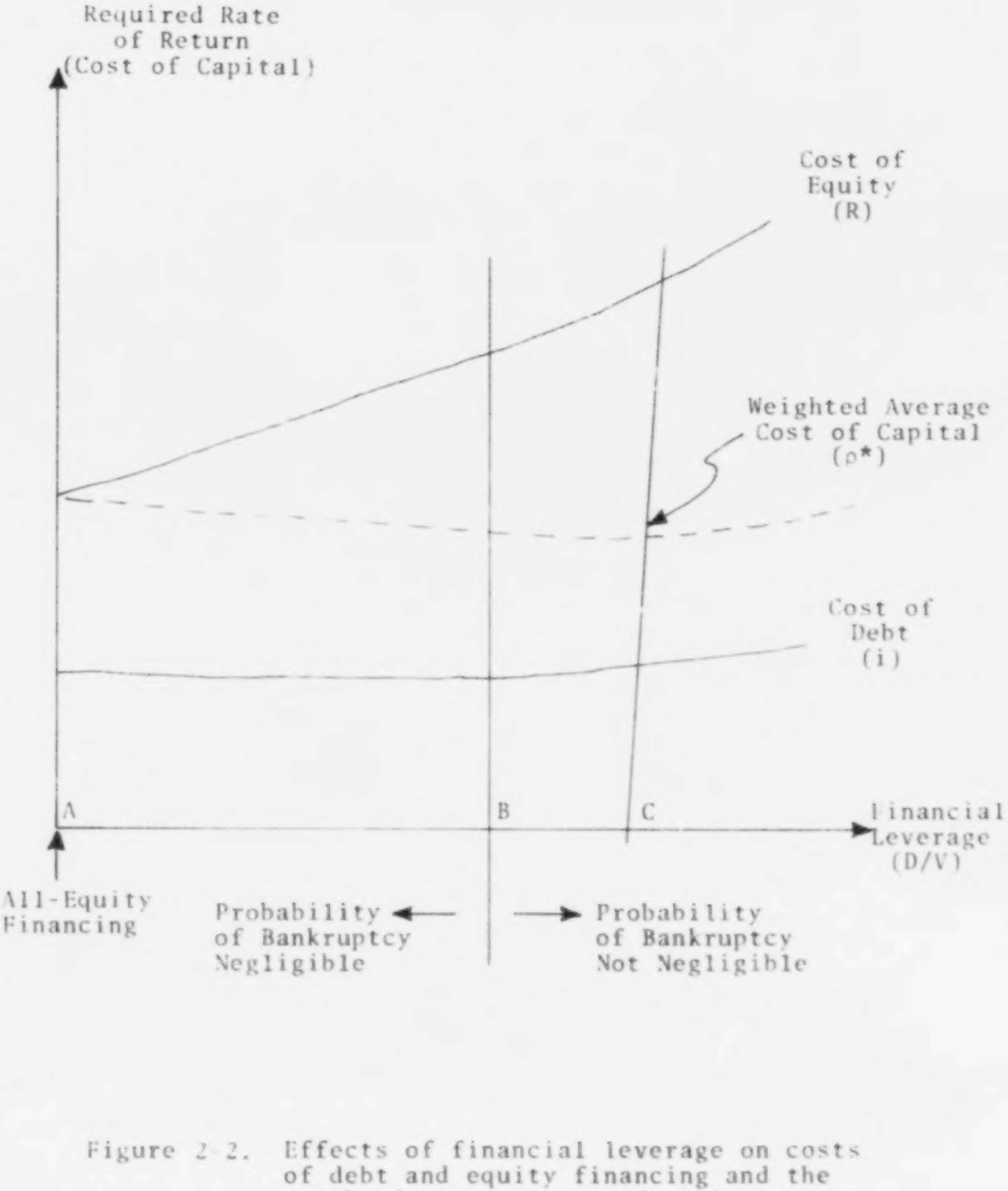
Required Rate
of Return
(Cost of Capital)
Cost of
Equity
(R)
Weighted Average
Cost of Capital
(ρ*)
Cost of
Debt
(i)
A
B
C
Financial
Leverage
(D/V)
All-Equity
Financing
Probability
of Bankruptcy
Negligible
Probability
of Bankruptcy
Not Negligible

Figure 2-2. Effects of financial leverage on costs of debt and equity financing and the weighted average cost of capital.

ρ^* is to determine the capitalized value of financial leases. This is done by capitalizing the expected lease payments at the corporate debt rate i.

The overall cost of capital is given by

$$\rho^* = i(1 - T)\frac{D}{V} + i(1 - T)\frac{L}{V} + k_p \cdot \frac{P}{V} + R \cdot \frac{E}{V} \qquad (2\text{-}28)$$

where

L = the capitalized value of financial leases

P = the market value of outstanding preferred stock

k_p = the yield on the preferred stock (i.e., the cost of capital for preferred stock)

V = D + L + P + E

All other symbols are as defined for Equation (2-27).

Example. Consider the case of the Winco Distribution Company. Winco has $5.3 million in long-term debt outstanding, $5.0 million of preferred stock, and $27.1 million of common stock (all figures are market values). Its current cost of borrowing is 5-1.4%, its preferred stock cost is 6%, and its current cost of equity capital is 10.6%. In addition, Winco has financial leases with annual payments of $850,000 per year (assumed to continue indefinitely` [illegible] italized value of the lease payments (at 5-1/4%) is $1[illegible] ion. The weighted average cost of capital using Equation (2-28) is 7%. The details of the calculation are given in Table 2-4.

Table 2-4

CALCULATION OF THE WEIGHTED AVERAGE COST OF CAPITAL: EXAMPLE

Source	Amount ($ Millions)	Proportion	After-Tax Cost	Weighted Cost
Long-term debt	5.3	0.099	2.625	0.26
Leases	16.2	0.302	2.625	0.79
Preferred Stock	5.0	0.093	6.000	0.56
Equity	27.1	0.506	10.600	5.36
Total	53.6	1.000		6.97

Of the variables used to compute the weighted average cost of capital, only R, the cost of equity capital, is not directly observable. (R was computed using Equation (2-8) with $\frac{D_1}{P_0} = 2.6\%$ and $g = 8\%$.)

The Effect of Debt Financing on the Estimation Equations for R

The estimation equations developed for R in Section II assumed the firm was all-equity financed. How will the form of these equations change when other types of financing are used as well?

The answer is, not at all, providing the proper assumptions are made. In Section II the use of a single value of R for all future periods assumed that the business risk[8] of the firm will remain stable over time, or else differing R's would be required to reflect changing business risks.[9] When debt (or other) financing is introduced, the same type of consideration is involved. The use of debt increases the equity risk (and thus increases R), but the same value of R for all future periods is appropriate as long as the financial risk of the firm is assumed to remain stable over time. Under these assumptions, the same discounted cash flow models developed in Section II can be used to estimate R for a firm which is only partially equity financed. Further, the estimating equations are not affected by the issue of new equity since we deal with per-share values. The necessary assumption is that new equity is issued at P_t, the per-share price of old equity.[10]

PART 2 — APPENDIX A

DEVELOPMENT OF SECURITY VALUATION MODELS

The basic valuation is given by

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + R)^t} \qquad (2-6)$$

Model 1—Perpetual Growth

Suppose that in each period the firm invests a fraction b of its earnings in new projects (note that $b < 1$ else the firm would never pay dividends). These projects produce a perpetual yield on equity of r, beginning with the next period. By definition, the earnings and dividends in period t are given by

$$E_t = E_{t-1} + rbE_{t-1}$$

$$= E_{t-1}(1 + br)$$

$$= E_0(1 + g)^t \qquad (2A-1)$$

where g is the growth rate of dividends per share ($g = br$)

$$D_t = D_0(1 + g)^t \qquad (2A-2)$$

Substituting Equation (2A-2) into Equation (2-6),

$$P_0 = \sum_{t=1}^{\infty} \frac{D_0(1 + g)^t}{(1 + R)^t}$$

$$= \frac{D_1}{R - g} \quad (g < R) \qquad (2A-3)$$

Note the condition that the growth rate must be less than the discount rate. If the product of the reinvestment rate r times the retention rate b equals R, P_0 will be infinite.

Model 2—No "Real Growth"

Suppose now that all equity reinvestment produces an expected return equal to the cost of capital R. Then the growth rate of per-share earnings and dividends will equal bR. (Note that g by definition will now be less than R, since $b < 1$.) Substituting $g = bR$ into Equation (2A-3)

$$P_0 = \frac{D_1}{R(1 - b)}$$

$$= \frac{E_1}{R} \qquad (2A-4)$$

Note that this equation also applies when $b = 0$, i.e., the firm pays out all of its earnings and does not grow.

Model 3—Finite Growth

Consider now a more realistic variant of the perpetual-growth model. The special investment opportunities $(r > R)$ are not available in perpetuity but only for some finite interval of T years.

The share value at t = 0 is equal to the no-real-growth value, plus the present value of the T years of real growth opportunities (PVGO).

$$P_0 = \frac{E_1}{R} + \text{PVGO} \qquad (2A\text{-}5)$$

In year t the firm invests I_t dollars per share (withheld earnings) which generates a perpetual return r. The value in period t of this investment (PVGO_t) is given by

$$\begin{aligned} \text{PVGO}_t &= \sum_{t=1}^{\infty} \frac{I_t \cdot r}{(1 + R)^t} - I_t \\ &= \frac{I_t \cdot r}{R} - I_t \\ &= \frac{I_t(r - R)}{R} \end{aligned} \qquad (2A\text{-}6)$$

The value at t = 0 of these T growth opportunities is given by

$$\text{PVGO} = \sum_{t=1}^{T} \frac{I_t(r - R)}{R(1 + R)^t} \qquad (2A\text{-}7)$$

Now

$$I_t = I_1(1 + g)^{t-1}$$

Therefore,

$$\begin{aligned} \text{PVGO} &= \frac{r - R}{R} \cdot I_1 \sum_{t=1}^{T} \frac{(1 + g)^{t-1}}{(1 + R)^t} \\ &= \frac{r - R}{R(1 + R)} I_1 \sum_{t=1}^{T} \frac{(1 + g)^{t-1}}{(1 + R)^{t-1}} \\ &= \frac{r - R}{R(1 + R)} \cdot I_1 \cdot \sum_{\tau=0}^{T-1} \frac{(1 + g)^{\tau}}{(1 + R)^{\tau}} \end{aligned} \qquad (2A\text{-}8)$$

It now remains to sum the terms in t. The sum of the geometric series with T - 1 terms (S_{T-1}) is given by

$$S_{T-1} = \frac{1 - \left(\frac{1+g}{1+R}\right)^T}{1 - \frac{1+g}{1+R}}$$

$$= \left[\frac{1 + R}{R - G}\right] \left[1 - \left(\frac{1 + g}{1 + R}\right)^T\right] \qquad (2A-9)$$

If, as we generally would expect, (1 + g)/(1 + R) is close to one, and T is not too large, then the right-hand term in Equation (2A-9) admits to a convenient approximation

$$\left(\frac{1 + g}{1 + R}\right)^T \cong 1 - T(g - R) \qquad (2A-10)$$

Substituting the expression for the sum (2A-9) and this approximation (2A-10) into Equation (2A-8) and simplifying, we obtain

$$PVGO = \frac{r - R}{R} \cdot I_1 \cdot T \qquad (2A-11)$$

We can now state the final form of Equation (2A-5)

$$P_0 = \frac{E_1}{R} + \left(\frac{r - R}{R}\right) \cdot I_1 \cdot T \qquad (2A-12)$$

FOOTNOTES FOR PART 2

1. This definition of the cost of capital is based on a definition by S. C. Myers in [19], pp. 63-65.

2. Precisely speaking, Equation (2-1) assumes that dividends are paid at the end of period T + 1. If dividends were paid and invested during the period, Equation (2-1) would have to be adjusted to allow for the additional return received. (See Part 3, Section III, item 2 for a detailed discussion of investment returns.) Whether the dividends are reinvested or consumed at the end of period T + 1 makes no difference in the return calculation. The effect will show up in the investment base for the next period (i.e., the stockholder will either have P_{t+1} or $P_{t+1} + D_{t+1}$ to invest in period t + 2).

3. Large portions of Section IV, including footnotes 4 - 6, have been reproduced verbatim from two unpublished manuscripts by Stewart C. Myers. These are, "Optimal Capital Structure" [23] and "Interaction of Investment and Financing Decisions" [24]. Both copyright 1975 by S. C. Myers. The excerpts from these manuscripts are included with the kind permission of the author.

4. The firm's taxable income is assumed to always exceed the interest payments. If this is not the case, the

exact present value of the tax savings becomes extremely elusive. One reason is the fact that accounting losses can be carried forward for tax purposes.

5. Actually, there are some problems with this assumption. What if the firm will issue more debt if earnings rise more than expected, and will retire debt if earnings are disappointing? Then the tax benefits are uncertain: they take on the risk characteristics of the firm's assets. However, it is difficult to specify precisely what the discount rate should be under these circumstances. Whatever the discount rate, the favorable tax treatment of debt provides a strong incentive for the use of debt financing.

6. These costs are discussed more fully in Robichek and Myers, "Problems in the Theory of Optimal Capital structure," *Journal of Finance and Quantitative Analysis*, June 1966, pp. 15-19. The existence of bankruptcy costs has been documented by Nevins D. Baxter: "Leverage, Risk of Ruin and the Cost of Capital," *Journal of Finance*, September 1967, p. 96.

7. This condition is actually more restrictive than necessary. What is really required is a rule for defining

ρ^* such that $NPV(\rho^*) > 0$ whenever $APV > 0$. However, this generalized requirement does not provide a practical procedure for estimating ρ^*, as does Equation (2-24).

8. The business risk of the firm refers to the riskiness of the firm's assets independently of how they are financed. For example, one can speak of the (business) risk of being in the electric utility or machine tool business. Financial risk refers to the additional risks borne by the stockholders when debt (or other funds of senior financing) is used by the firm. Since the stockholders have a residual claim to the firm's earnings (and assets), the use of debt will tend to magnify the business risk of the firm as it is passed along to the stockholders. This magnification is referred to as financial risk. An example of this effect is shown in Table 2-2.

9. To be precise, the use of a constant discount rate R assumes that the risk of the firm's dividend stream increases at a constant rate as a function of time. That is, risk increases at a constant rate as one looks further and further into the future. This is not an unreasonable assumption, and is almost universally

assumed in studies using discounted cash flow valuation models. For a fuller discussion of this matter, see Robichek and Myers, "Conceptual Problems in the use of Rick-Adjusted Discount Rates," *The Journal of Finance*, Vol. XXI (Dec. 1966), pp. 727-730.

10. This is precisely true only in the absence of equity issue costs. From the point of view of the existing stockholders, the required rate of return on new issues must be slightly higher than R to offset the effect of transactions costs. This will allow the new stockholders to earn the rate R on the gross proceeds of the issue (i.e., their investment).

ESTIMATION OF THE COST OF CAPITAL
FOR MAJOR UNITED STATES INDUSTRIES
WITH APPLICATION TO POLLUTION-CONTROL INVESTMENTS

PART 3. THEORY: RISK AND RETURN CONCEPTS

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November 1975

I. INTRODUCTION

As discussed in Part 2, the cost of equity capital is the rate of return (dividends plus capital gains) investors expect to earn by holding a firm's common stock. Assuming that investors are by-and-large averse to bearing risk, the required rate of return will depend on the stock's risk level: The higher the risk, the higher the expected rate of return (cost of capital).

But this definition of the cost of capital raises as many questions as it answers. How is risk to be defined and measured? What are the current theories about the relationship between risk and required returns? How are stock market (i.e., financial) risk measures related to the characteristics of firms' (real) assets, such as the variability of corporate earnings, the degree of financial leverage, and the growth rate of earnings?

These questions are the subject of Part 3. Part 3 is organized in four sections. Sections II and III provide a nontechnical introduction to the current risk and return theories. Section IV summarizes the existing empirical studies relating to the real determinants of a stock's financial risk.

Sections II and III perform overlapping functions. Section II provides a brief introduction to the "high points" of the theory. Section III presents a detailed survey for the reader intent on a greater depth of understanding. It contains a two-part article written by Professors Franco Modigliani and Gerald Pogue entitled, "An Introduction to Risk and Return: Concepts and Evidence," [16]. Readers familiar with the basic notions of portfolio and capital markets theories could well skip Sections II and III.

II. A BRIEF INTRODUCTION TO THE THEORY OF RISK AND THE REQUIRED RATE OF RETURN[1]

(a) The Risk of Individual Common Stocks

The measurement of common stock risk is a complex subject. No-one has "all the answers". Yet enough is known and reasonably well agreed upon to identify the main elements of stock risk and to measure them using past data. The purpose of this section is to explain this methodology.

The Risk of Holding a Portfolio of Securities

Some investors hold stock in only one company, others in hundreds of companies. But in any case, they are interested in the rate of return realized on their portfolios, not the rate of return on any individual security *per se*.

Let R_p stand for the rate of return realized on an investor's portfolio over a given period of time. R_p is equal to dividends plus capital gains, divided by the value of the portfolio at the start of the period:

$$R_p = \frac{D_p + \Delta V_p}{V_p} \qquad (3\text{-}1)$$

where

D_p = dividends received from the portfolio

V_p = market value of the portfolio at the start of the period

ΔV_p = capital gain on the portfolio during the period

The investor is obviously concerned with the predictability of R_p—i.e., with the possible difference between the anticipated and the actual rate of return. The usual statistical measure of the extent of possible deviation is σ_p, the standard deviation of R_p.

Diversification

Intelligent diversification can substantially reduce the risk of a portfolio return. But the risk cannot be eliminated entirely.

Wagner and Lau [28] have reported an experiment which clearly illustrates this. They formed portfolios of varying size by randomly selecting stocks from a sample of firms rated A+ in Standard and Poor's 1960 Earnings and Dividend Rankings. Then they determined what the month-by-month portfolio returns would have been during the 1960-70 period, and calculated the standard deviation of this monthly return. The results are shown in Table 3-1.

The reduction in risk due to diversification is dramatic. However, it is also evident that additional diversification

Table 3-1

RISK VERSUS DIVERSIFICATION:

RANDOMLY SELECTED PORTFOLIOS*

Number of Securities in Portfolio	Standard Deviation of Return— Percent/Month	Correlation with the Return on a Market Index**
1	7.0	0.54
2	5.0	0.63
3	4.8	0.75
4	4.6	0.77
5	4.6	0.79
10	4.2	0.85
15	4.0	0.88
20	3.9	0.89

* Wagner and Lau examined ten distinct portfolios for each level of diversification. Thus the 3.9 percent standard deviation at the 20-security diversification level is the average standard deviation of 10 randomly selected 20-security portfolios.

** The index in this case was the average return on all N. Y. Stock Exchange stocks.

Source: Wagner and Lau [28].

yields a rapidly diminishing reduction in risk. The improvement is slight when the number of securities is increased beyond, say, ten.

Note that the return on a diversified portfolio "follows the market" very closely. The 20-security portfolios had a correlation of 0.89 with the market (perfect positive correlation results in a coefficient of 1.0). The implication is that the risk remaining in the 20-security portfolio is predominantly a reflection of uncertainty about the performance of the stock market in general.

Thus, one is led to the distinction between a security's unsystematic risk, which can be eliminated by diversification, and its systematic risk, which cannot. The relationship between the two is illustrated in Figure 3-1.

The systematic risk is due to the fact that the return on nearly every stock depends to some degree on the overall performance of the stock market. Investors are thus exposed to "market uncertainty" no matter how many stocks they hold. Consequently, the returns on diversified portfolios are highly correlated with the market.

The Risk of Individual Securities

Since the investor is interested in the risk of his portfolio, he will judge the risk of any security by assessing its contribution to the portfolio risk.

However, the nature of this contribution depends on how diversified the portfolio is. At one extreme is the investor who holds only one stock. In this instance, the portfolio risk can be measured by the standard deviation of the stock's return.

At the other extreme is the investor who holds "the market", or a portfolio sufficiently diversified that its

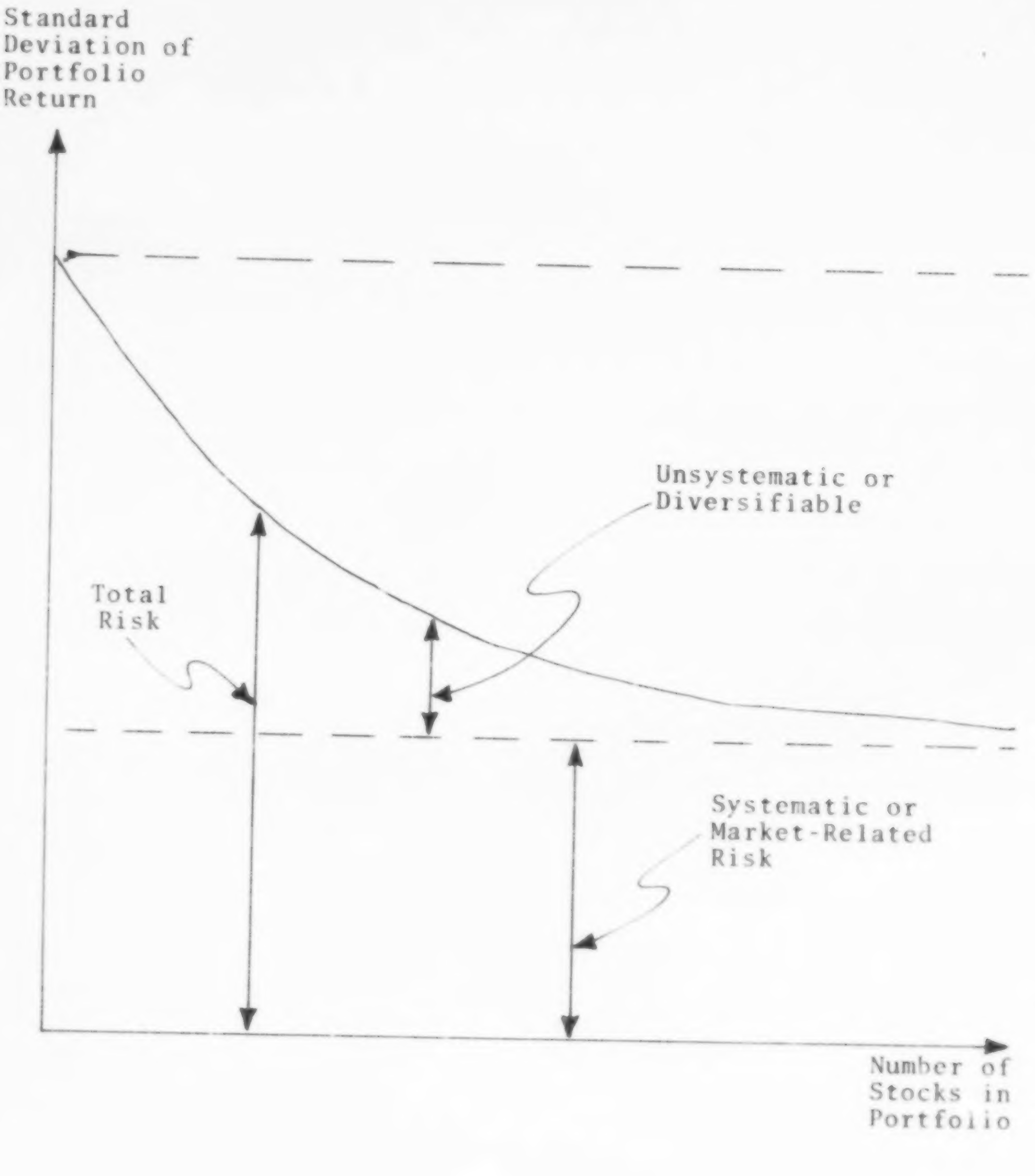
Standard Deviation of Portfolio Return
Unsystematic or Diversifiable
Total Risk
Systematic or Market-Related Risk
Number of Stocks in Portfolio

Figure 3-1. Systematic versus unsystematic risk.

return is highly correlated with the market. In this case, standard deviation is not a good risk measure since some fluctuations in the stock's return will tend to "cancel out" against fluctuations in the returns on other securities. A stock that seems highly risky to an undiversified investor may contribute very little to the risk of a diversified portfolio.

Obviously, the problem is to measure a stock's systematic risk—that part of its risk that cannot be diversified away.

The key to the measurement is the fact that the risk of a diversified portfolio is predominantly a reflection on uncertainty about the performance of the market. Thus, in measuring systematic risk we can concentrate on the extent to which individual securities' returns depend on market performance.

This is why a stock's systematic risk is measured by its beta (β), sometimes called the "market sensitivity index". Beta can be thought of as the slope of a line fitted to a plot of rates of return on the stock versus rates of return on a "market portfolio" composed of all stocks. This is shown in Figure 3-2.

The hypothetical security shown in Figure 3-2 has a beta of 1. The meaning of this is that, if the actual return on the market is 10% in excess of expectations, the stock's return will tend to be 10% above expectations. To put it n

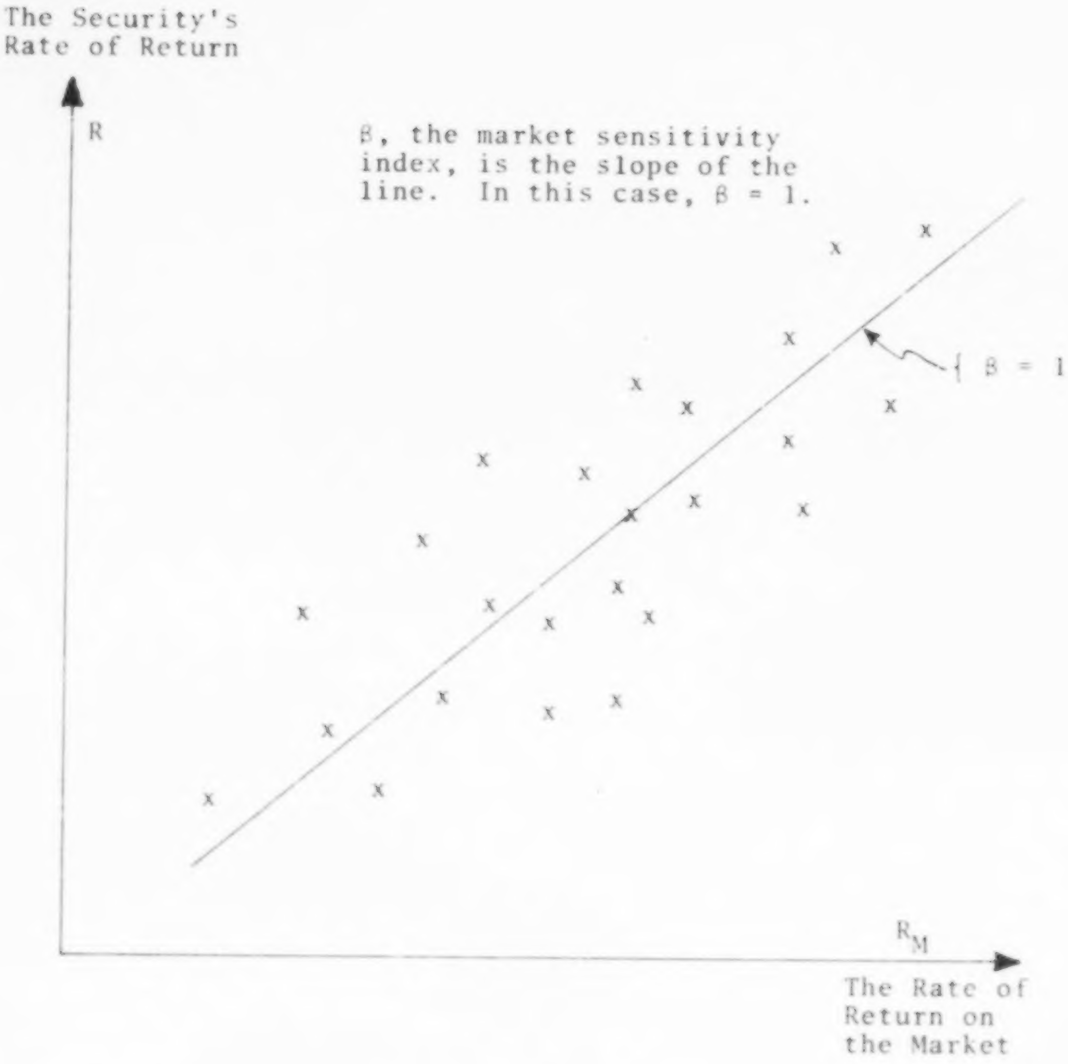

Figure 3-2. Calculation of a security's market sensitivity index from past data.

every-day terms, the price of a stock with $\beta = 1$ will tend to rise 10% when the market rises 10%, and fall 10% when the market falls 10%. An average stock will have a beta of 1.0.

Consider a well-diversified portfolio made up of securities having $\beta = 2$. The value of such a portfolio will tend to go up 20% when the market goes up 10%. On the other hand, if the market falls, the portfolio's value will fall by double the amount. Such a portfolio will have twice the standard deviation of the market portfolio, and four times the standard deviation of a portfolio composed of stocks with $\beta = 0.5$.

In other words, the standard deviation of a well-diversified portfolio is approximately proportional to the portfolio's beta. Any portfolio's beta is simply a weighted average of the betas of the securities in the portfolio. Therefore, we can take a stock's beta as a measure of the stock's contribution to portfolio risk, assuming the portfolio is well diversified.

The analysis so far can be summarized as follows:

1. The risk of a portfolio can be measured by the standard deviation of its rate of return.
2. The risk of an individual security is its contribution to portfolio risk.
3. The standard deviation of a stock's return is the relevant measure of risk for the undiversified investor.

4. However, a stock's standard deviation partly reflects unsystematic risk—risk that can be eliminated by diversification. Only the systematic component of stock risk is relevant to the well-diversified investor.
5. A stock's systematic risk is measured by its beta, or "market sensitivity index".

Needless to say, there are investors holding only a handful of stocks who cannot be described as either undiversified or well diversified. A stock's risk to such an investor will depend on both its standard deviation and its beta.

Since most investors have extensive opportunities for diversification (even the proverbial widow or orphan can buy a mutual fund), beta is usually considered the more relevant risk measure.

Measuring Betas

Obviously there is no risk in hindsight. The investor is worried about the unpredictability of the future return on his portfolio, and in individual stocks' contributions to this unpredictability.

It is not feasible to obtain direct measurements of how investors assess the risk of various stocks at this point in time. But these assessments will certainly depend on past experience. A stock's past behavior can provide strong evidence pertaining to its current and future risk.

The basic data for estimating betas are past rates of return earned over relatively short intervals—usually weeks or months. For example, the beta calculations in Part 4 are based on monthly rates of return which occurred during the period February 1962 to December 1974.

Beta is calculated by fitting a straight line to a plot of observed stock returns versus observed returns on the market (see Figure 3-2). The return on the market as a whole is measured by a broadly based market index such as the Standard and Poor's Composite 500 Stock Index.

The equation of the fitted line is

$$R_j = \hat{\alpha}_j + \hat{\beta}_j R_M + \hat{\varepsilon}_j \qquad (3-2)$$

where

$\hat{\alpha}_j$ = the intercept of the fitted line

$\hat{\varepsilon}_j$ = the variation around the fitted line

$\hat{\beta}_j$ = stock j's systematic risk

It is customary to put a hat (ˆ) over the estimated values α_j, β_j, and ε_j. It is important to remember that these estimated values may differ from the true values because of statistical difficulties. However, the extent of possible error can be measured, providing a range within which the true value is almost certain to lie.

(b) Basic Risk-Return Concepts: The Capital Asset Pricing Model

In the past several years a great deal of empirical research has been conducted on the relationship between risk and rate of return in United States capital markets. Several dozen articles have been published, representing diverse points of view. However, there is a common theme in these studies: over long periods of time, higher risk securities have, on the average, achieved higher rates of return.

The Capital Asset Pricing Model (CAPM)

The typical starting point for current research in this area is the "capital asset pricing model". According to the model, $E(R_j)$, the rate of return expected by shareholders of the j^{th} firm, is equal to the rate on a risk-free asset (R_f) plus a risk-premium which is proportional to the security's systematic risk (β_j). That is,

$$E(R_j) = R_f + \beta_j \cdot [E(R_M) - R_f] \qquad (3\text{-}3)$$

The constant of proportionality, $[E(R_M) - R_f]$, is the expected risk premium on the market portfolio. The market portfolio could be represented by the Standard and Poor's 500 Stock Composite Index, for example, and the risk-free rate by yields on short-term treasury bills.

The predictions of the model are inherently sensible. For safe investments ($\beta_j = 0$), the model predicts that investors would expect to earn the risk-free rate of interest.

For a risky investment ($\beta_j > 0$) investors would expect a rate of return proportional to the stock's beta. Thus stocks with lower-than-average betas (such as most utilities) would offer expected returns less than the expected market return. Stocks with above-average values of beta (such as most airline securities) would offer expected returns in excess of the market.

Tests of the Capital Asset Pricing Model

If the capital asset pricing model is right, the empirical tests should show the following:

1. On the average, and over long periods of time, the securities with high systematic risk should have high rates of return.
2. On the average, there should be a linear relationship between systematic risk and return.
3. Unsystematic risk should play no significant role in explaining differences in security returns.

These predictions have been tested in several recent statistical studies. One such study was by Black, Jensen, and Scholes [3]. Their results showed that, over the 35-year period from 1931 to 1965, average stock returns increased with increasing betas, but not as much as predicted by the capital asset pricing model. Also, their results indicated that there was little reason to question the linearity of the relationship over the test period. Figure 3-3 shows a plot of

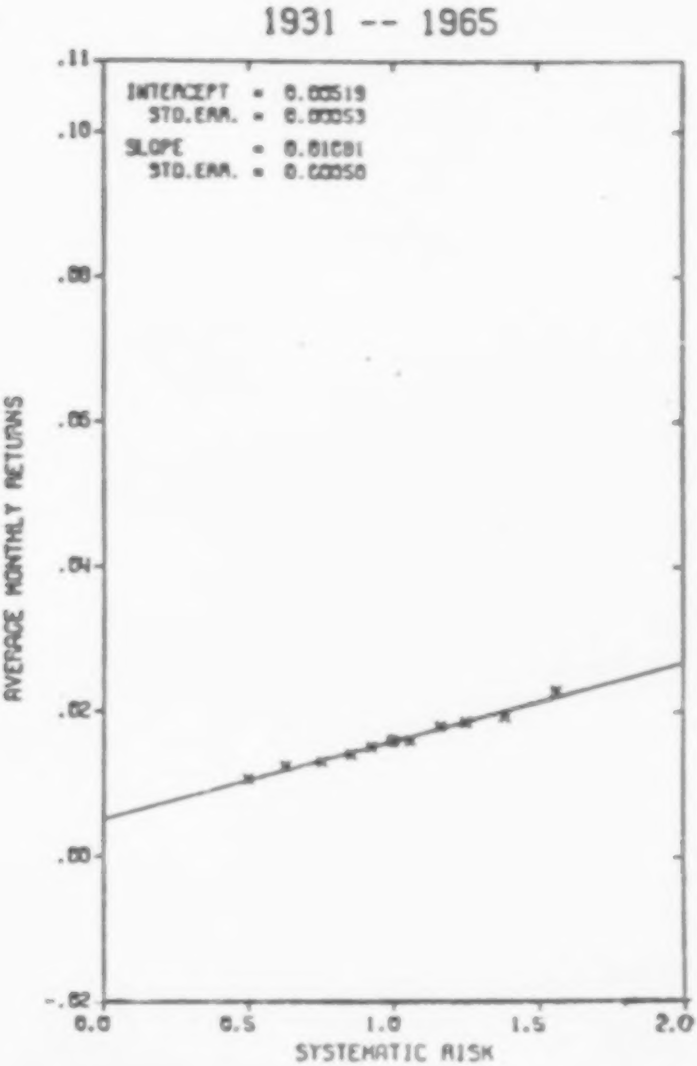

Figure 3-3. Results of Black, Jensen and Scholes Study—Average monthly returns versus systematic risk for the 35-year period 1931-1965 for ten portfolios and the market portfolio (Black, Jensen and Scholes [3], Figure 7, p. 104).

average month returns for the Black, Jensen and Scholes test portfolios versus their systematic risk.

Briefly, the major result of this and other studies can be summarized as follows:

1. The evidence shows a significant positive relationship between realized returns and systematic risk. However, the relationship is not always as strong as predicted by the capital asset pricing model.
2. The relationship between risk and return appears to be linear. The studies give no evidence of significant curvature in the risk-return relationship.
3. Tests which attempt to discriminate between the effects of systematic and unsystematic risk do not yield definitive results. Both kinds of risk appear to be positively related to security returns. However, the relationship between return and unsystematic risk is at least partly spurious—that is, partly reflecting statistical problems rather than the true nature of capital markets.

Obviously, one cannot claim that the capital asset pricing model is absolutely right. On the other hand, the empirical tests do support the view that beta is a useful risk measure and that investors in high beta stocks expect correspondingly high rates of return.

The Two-Factor Model

The difficulty with the capital asset pricing model is that the observed tradeoff between return and risk is smaller than predicted. In the Black, Jensen and Scholes tests, for example, the average return during the 35-year test period increased by approximately 1.08 percent per month for a one-unit increase in beta; this is only about three quarters of the amount predicted by the capital asset pricing model.

Black [2] has proposed a variant of the CAPM known as the two factor model. This model results from a somewhat different theoretical reason but the result is structurally similar. In the two-factor model, the risk-free rate of the CAPM is replaced by the expected return on a "zero-beta" portfolio, designated ER_z. The model is given by

$$ER_j = ER_z + \beta_j (ER_M - ER_z) \qquad (3\text{-}4)$$

The R_z factor represents the expected return on a portfolio whose returns are uncorrelated with the market. This model conforms to the empirical evidence much more closely than the CAPM. However, the theoretical nature of the R_z factor is less well understood.

III. RISK AND THE REQUIRED RATE OF RETURN*

by Franco Modigliani and Gerald A. Pogue

An Introduction to Risk and Return

Concepts and Evidence

1. Introduction

Portfolio theory deals with the selection of optimal portfolios by rational risk-averse investors: that is, by investors who attempt to maximize their expected portfolio returns consistent with individually acceptable levels of portfolio risk. Capital markets theory deals with the implications for security prices of the decisions made by these investors: that is, what relationship should exist between security returns and risk if investors behave in this optimal fashion. Together, portfolio and capital markets theories provide a framework for the specification and measurement of investment risk, for developing relationships between expected security return and risk, and for measuring the performance of managed portfolios such as mutual funds and pension funds.

The purpose of this article is to present a nontechnical introduction to portfolio and capital markets theories. Our hope is to provide a wide class of readers with an understanding of the foundation upon which the modern risk and performance measures are based, by presenting the main elements of the theory along with the results of some of the more important empirical tests. We are attempting to present not an exhaustive survey of the theoretical and empirical literature, but rather the main thread of the subject leading the reader from the most basic concepts to the more sophisticated but practically useful results of the theory.

2. Investment Return

Measuring historical rates of return is a relatively straightforward matter. We will begin by showing how investment return during a single interval can be measured, and then present three commonly used measures of average return over a series of such intervals.

The return on an investor's portfolio during a given interval is equal to the change in value of the portfolio plus any distributions received from the portfolio expressed as a fraction of the initial portfolio value. It is important that any capital or income distributions made to the investor be included, or else the measure of return will be deficient. Equivalently, the return can be thought of as the amount (expressed as a fraction of the initial portfolio value) that can be withdrawn at the end of the interval while maintaining the principal intact. The return on the investor's portfolio, designated R_p, is given by

$$R_p = \frac{V_1 - V_0 + D_1}{V_0}, \qquad (1)$$

where

V_1 = the portfolio market value at the end of the interval
V_0 = the portfolio market value at the beginning of the interval
D_1 = cash distributions to the investor during the interval

The calculation assumes that any interest or dividend income received on the portfolio securities and not distributed to the investor is reinvested in the portfolio (and thus reflected in V_1). Furthermore, the calculation assumes that any

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distributions occur at the end of the interval, or are held in the form of cash until the end of the interval. If the distributions were reinvested prior to the end of the interval, the calculation would have to be modified to consider the gains or losses on the amount reinvested. The formula also assumes no capital inflows during the interval. Otherwise, the calculation would have to be modified to reflect the increased investment base. Capital inflows at the end of the interval, however, can be treated as just the reverse of distributions in the return calculation.

Thus given the beginning and ending portfolio values, plus any contributions from or distributions to the investor (assumed to occur at the end of the interval), we can compute the investor's return using Equation (1a). For example, if the XYZ pension fund had a market value of $100,000 at the end of June, capital contributions of $10,000, benefit payments of $5,000 (both at the end of July), and an end-of-July market value of $95,000, the return for the month is a loss of 10 per cent.

The arithmetic average return is an unweighted average of the returns achieved during a series of such measurement intervals. For example, if the portfolio returns [as measured by Equation (1a)] were -10 per cent, 20 per cent, and 5 per cent in July, August, and September respectively, the average monthly return is 5 per cent. The general formula is

$$R_A = \frac{R_{P1} + R_{P2} + \dots + R_{PN}}{N}, \qquad (1b)$$

where

R_A = the arithmetic average return

R_{PK} = the portfolio return in interval k, k=1, . . ., N

N = the number of intervals in the performance-evaluation period

The arithmetic average can be thought of as the mean value of the withdrawals (expressed as a fraction of the initial portfolio value) that can be made at the end of each interval while maintaining the principal intact. In the above example, the investor must add 10 per cent of the principal at the end of the first interval and can withdraw 20 per cent and 5 per cent at the end of the second and third, for a mean withdrawal of 5 per cent of the initial value per period.

The time-weighted return measures the compound rate of growth of the initial portfolio during the performance-evaluation period, assuming that all cash distributions are reinvested in the portfolio. It is also commonly referred to as the "geometric" rate of return. It is computed by taking the geometric average of the portfolio returns computed from Equation (1a). For example, let us assume the portfolio returns were -10 per cent, 20 per cent, and 5 per cent in July, August, and September, as in the example above. The time-weighted rate of return is 4.3 per cent per month. Thus one dollar invested in the portfolio at the end of June would have grown at a rate of 4.3 per cent per month during the three-month period. The general formula is

$$R_T = [(1 + R_{P1})(1 + R_{P2}) \dots (1 + R_{PN})]^{1/N} - 1, \qquad (1c)$$

where

R_T = the time-weighted rate of return

R_{PK} = the portfolio return during the interval k, k=1, . . ., N

N = the number of intervals in the performance-evaluation period

In general, the arithmetic and time-weighted average returns do not coincide. This is because, in computing the arithmetic average, the amount invested is assumed to be maintained (through additions or withdrawals) at its initial value. The time-weighted return, on the other hand, is the return on a portfolio that varies in size because of the assumption that all proceeds are reinvested. The failure of the two averages to coincide is illustrated in the following example. Consider a portfolio with a $100 market value at the end of 1972, a $200 value at the end of 1973, and a $100 value at the end of 1974. The annual returns are 100 per cent and -50 per cent. The arithmetic and time-weighted average returns are 25 per cent and zero per cent respectively. The arithmetic average return consists of the average of $100 withdrawn at the end of Period 1, and $50 replaced at the end of Period 2. The compound rate of return is clearly zero, the 100 per cent return in the first period being exactly offset by the 50 per cent loss in the second period on the larger asset base. In this example the arithmetic average exceeded the time-weighted average return. This always proves to be true, except in the special situation where the returns in each interval are the same, in which case the averages are identical.

The dollar-weighted return measures the average rate of growth of all funds invested in the portfolio during the performance-evaluation period—that is, the initial value plus any contributions less any distributions. As such, the rate is influenced by the timing and magnitude of the contributions and distributions to and from the portfolio. The measure is also commonly referred to as the "internal rate of return." It is important to corporations, for example, for comparison with the actuarial rates of portfolio growth assumed when funding their employee pension plans.

The dollar-weighted return is computed in exactly the same way that the yield to maturity on a bond is determined. For example, consider a portfolio with market value of \$100,000 at the end of 1973 (V_0), capital withdrawals of \$5,000 at the end of 1974, 1975, and 1976 (C_1, C_2, and C_3), and a market value of \$110,000 at the end of 1976 (V_3). Using compound interest tables, the dollar-weighted rate of return is found by trial and error to be 8.1 per cent per year during the three-year period. Thus each dollar in the fund grew at an average rate of 8.1 per cent per year. The formula used is

$$V_0 = \frac{C_1}{(1+R_D)} + \frac{C_2}{(1+R_D)^2} + \frac{C_3}{(1+R_D)^3} + \frac{V_3}{(1+R_D)^3} \qquad (1d)$$

where

R_D = the dollar weighted rate of return

What is the relationship between the dollar-weighted return (internal rate of return) and the previously defined time-weighted rate of return? It is easy to show that under certain special conditions both rates of return are the same. Consider, for example, a portfolio with initial total value V_0. No further additions or withdrawals occur and all dividends are reinvested. Under these special circumstances all of the C's in Equation (1d) are zero so that

$$V_0 = \frac{V_0(1+R_{p1})(1+R_{p2})(1+R_{p3})}{(1+R_D)^3}$$

where R_p's are the single-period returns. The numerator of the expression on the right is just the value of the initial investment at the end of the three periods (V_3). Solving for R_D we find

$$R_D = [(1+R_{p1})(1+R_{p2})(1+R_{p3})]^{1/3} - 1 ,$$

which is the same as the time-weighted rate of return R_T given by Equation (1c). However, when contributions or withdrawals to the portfolio occur, the two rates of return are not the same. Because the dollar-weighted return (unlike the time-weighted return) is affected by the magnitude and timing of portfolio contributions and distributions (which are typically beyond the portfolio manager's control), it is not useful for measuring the investment performance of the manager. For example, consider two identical portfolios (designated A and B) with year-end 1973 market values of \$100,000. During 1974 each portfolio has a 20 per cent return. At the end of 1974, portfolio A has a capital contribution of \$50,000 and portfolio B a withdrawal of \$50,000. During 1975, both portfolios suffer a 10 per cent loss resulting in year-end market values of \$153,000 and \$63,000 respectively. Now, both portfolio managers performed equally well, earning 20 per cent in 1974 and -10 per cent in 1975, for a time-weighted average return of 3.9 per cent per year. The dollar-weighted returns are not the same, however, due to the different asset bases for 1975, equaling 1.2 per cent and 8.2 per cent for portfolios A and B respectively. The owners of portfolio B, unlike those of A, made a fortuitous decision to reduce their investment prior to the 1975 decline.

In the remainder of this article, when we mention rate of return, we will generally be referring to the single interval measure given by Equation (1a). However, from time to time we will refer to the arithmetic and geometric averages of these returns.

3. Portfolio Risk

The definition of investment risk leads us into much less well explored territory. Not everyone agrees on how to define risk, let alone how to measure it. Nevertheless, there are some attributes of risk which are reasonably well accepted.

If an investor holds a portfolio of treasury bonds, he faces no uncertainty about monetary outcome. The value of the portfolio at maturity of the notes will be identical with the predicted value. In this case the investor bears no monetary risk. However, if he has a portfolio composed of common stocks, it will be impossible to exactly predict the value of the portfolio as of any future date. The best he can do is to make a best guess or most likely estimate, qualified by statements about the range and likelihood of other values. In this case, the investor does bear risk.

One measure of risk is the extent to which the *future* portfolio values are likely to diverge from the expected or predicted value. More specifically, risk for most investors is related to the chance that future portfolio values will be less than expected.

Thus if the investor's portfolio has a current value of $100,000, and an expected value of $110,000 at the end of the next year, he will be concerned about the probability of achieving values less than $110,000.

Before proceeding to the quantification of risk, it is convenient to shift our attention from the terminal value of the portfolio to the portfolio rate of return, R_p, since the increase in portfolio value is directly related to R_p.[1]

A particularly useful way to quantify the uncertainty about the portfolio return is to specify the probability associated with each of the possible future returns. Assume, for example, that an investor has identified five possible outcomes for his portfolio return during the next year. Associated with each return is a subjectively determined probability, or relative chance of occurrence. The five possible outcomes are:

Possible Return	Subjective Probability
50%	0.1
30%	0.2
10%	0.4
-10%	0.2
-30%	0.1
	1.00

Note that the probabilities sum to 1.00 so that the actual portfolio return is confined to take one of the five possible values. Given this probability distribution, we can measure the expected return and risk for the portfolio.

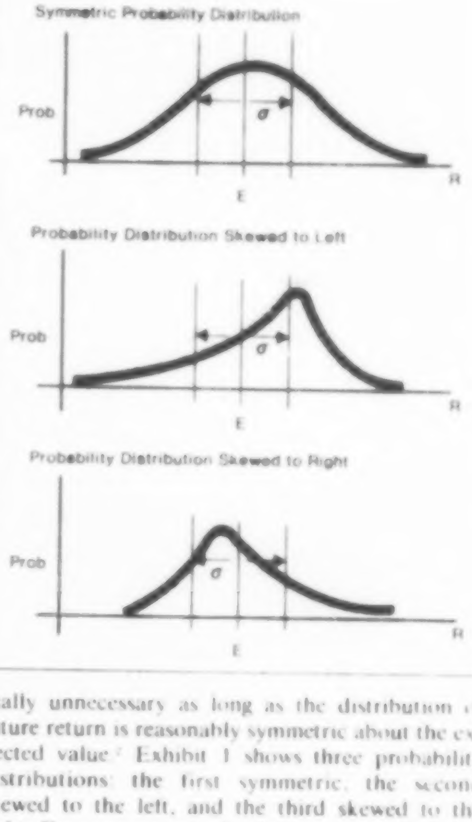
The expected return is simply the weighted average of possible outcomes, where the weights are the relative chances of occurrence. The expected return on the portfolio is 10 per cent, given by:

$$\begin{aligned} E(R_p) &= \sum_{i=1}^{5} P_i \; R_i \\ &= 0.1\,(50.0) + 0.2\,(30.0) + 0.4\,(10.0) \\ &\quad + 0.2\,(-10.0) + 0.1\,(-30.0) \\ &= 10\%, \end{aligned} \tag{2}$$

where the R_i's are the possible returns and the P_i's the associated probabilities.

If risk is defined as the chance of achieving returns less than expected, it would seem to be logical to measure risk by the dispersion of the possible returns below the expected value. However, risk measures based on below-the-mean variability are difficult to work with and are actually unnecessary as long as the distribution of future return is reasonably symmetric about the expected value.[2] Exhibit 1 shows three probability distributions: the first symmetric, the second skewed to the left, and the third skewed to the right. For a symmetric distribution, the dispersion of returns on one side of the expected return is the same as the dispersion on the other side.

EXHIBIT 1
POSSIBLE SHAPES FOR
PROBABILITY DISTRIBUTIONS



Empirical studies of realized rates of return on diversified portfolios show that skewness is not a significant problem.[3] If future distributions are shaped like historical distributions, then it makes little difference whether we measure variability of returns on one or both sides of the expected return. If the probability distribution is symmetric, measures of the total variability of return will be twice as large as measures of the portfolio's variability below the expected return. Thus if total variability is used as a risk surrogate, the risk rankings for a group of portfolios will be the same as when variability below the expected return is used. It is for this reason that total variability of

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1. Footnotes appear at end of article.

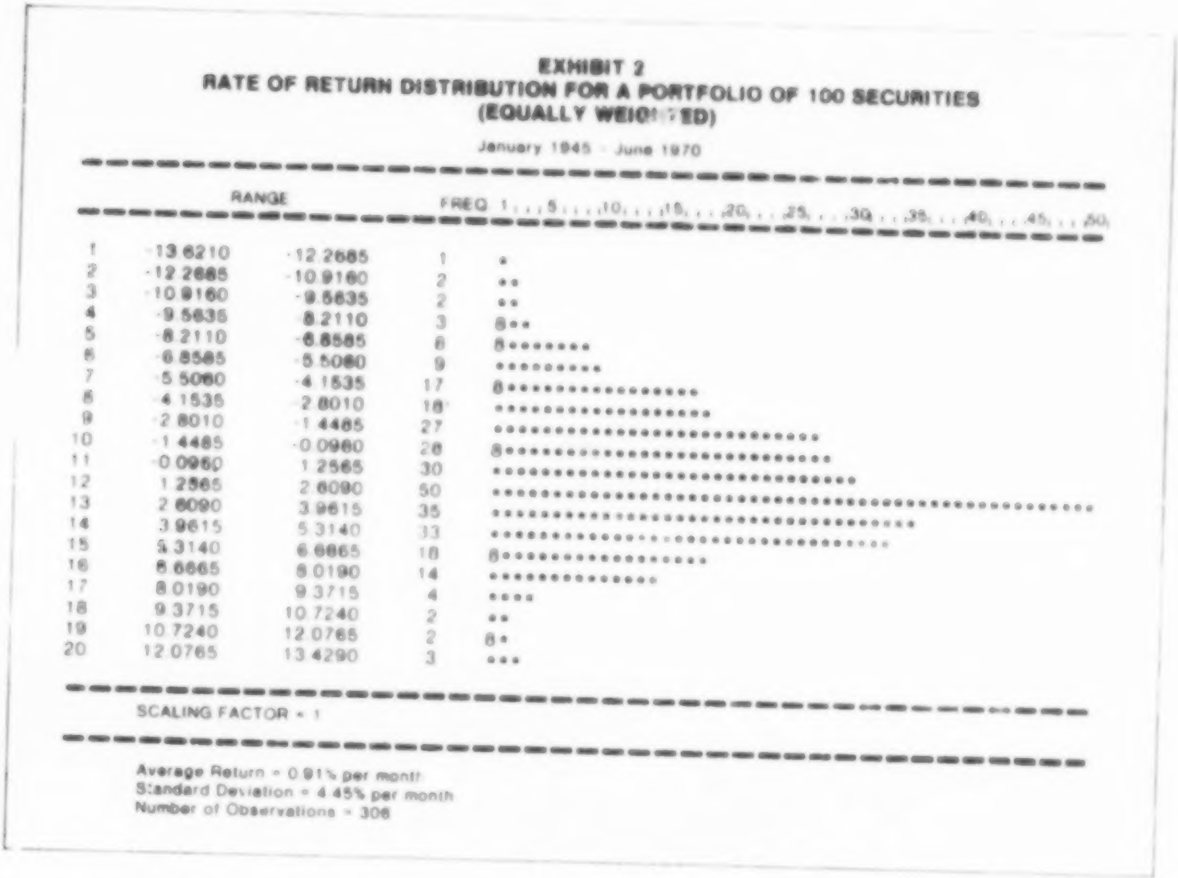

EXHIBIT 2
RATE OF RETURN DISTRIBUTION FOR A PORTFOLIO OF 100 SECURITIES
(EQUALLY WEIGHTED)

January 1945 - June 1970

```
----------------------------------------------------------------------------
           RANGE                 FREQ 1...5....10....15...20...25...30...35...40...45...50
----------------------------------------------------------------------------
 1    -13.6210    -12.2685     1    *
 2    -12.2685    -10.9160     2    **
 3    -10.9160     -9.5635     2    **
 4     -9.5635     -8.2110     3    8**
 5     -8.2110     -6.8585     8    8*******
 6     -6.8585     -5.5060     9    *********
 7     -5.5060     -4.1535    17    8****************
 8     -4.1535     -2.8010    18    ******************
 9     -2.8010     -1.4485    27    ***************************
10     -1.4485     -0.0960    28    8***************************
11     -0.0960      1.2565    30    ******************************
12      1.2565      2.6090    50    **************************************************
13      2.6090      3.9615    35    ***********************************
14      3.9615      5.3140    33    *********************************
15      5.3140      6.6665    18    8*****************
16      6.6665      8.0190    14    **************
17      8.0190      9.3715     4    ****
18      9.3715     10.7240     2    **
19     10.7240     12.0765     2    8*
20     12.0765     13.4290     3    ***
----------------------------------------------------------------------------
       SCALING FACTOR = 1
----------------------------------------------------------------------------
       Average Return = 0.91% per montl
       Standard Deviation = 4.45% per month
       Number of Observations = 306
```

returns has been so widely used as a surrogate for risk.

It now remains to choose a specific measure of total variability of returns. The measures most commonly used are the variance and standard deviation of returns.

The variance of return is a weighted sum of the squared deviations from the expected return. Squaring the deviations ensures that deviations above and below the expected value contribute equally to the measure of variability, regardless of sign. The variance, designated σ_p^2 for the portfolio in the previous example is given by

$$\begin{aligned}\sigma_p^2 &= \sum_{i=1}^{5} P_i \left(R_i - E(R_p)\right)^2 \\ &= 0.1(50.0 - 10.0)^2 + 0.2(30.0 - 10.0)^2 \\ &\quad + 0.4(10.0 - 10.0)^2 + 0.2(-10.0 - 10.0)^2 \\ &\quad + 0.1(-30.0 - 10.0)^2 \\ &= 480 \text{ per cent squared} \qquad (3)\end{aligned}$$

The standard deviation (σ_p) is defined as the square root of the variance. It is equal to 22 per cent. The larger the variance or standard deviation, the greater the possible dispersion of future realized values around the expected value, and the larger the investor's uncertainty. As a rule of thumb for symmetric distributions, it is often suggested that roughly two-thirds of the possible returns will lie within one standard deviation either side of the expected value, and that 95 per cent will be within two standard deviations.

Exhibit 2 shows the historical return distributions for a diversified portfolio. The portfolio is composed of approximately 100 securities, with each security having equal weight. The month-by-month returns cover the period from January 1945 to June 1970. Note that the distribution is approximately, but not perfectly, symmetric. The arithmetic average return for the 306-month period is 0.91 per cent per month. The standard deviation about this average is 4.45 per cent per month.

EXHIBIT 3
RATE OF RETURN DISTRIBUTION FOR NATIONAL DEPARTMENT STORES

January 1945 – June 1970

```
---------------------------------------------------------------------------------------------
                RANGE                FREQ  IIII5IIII10IIII15IIII20IIII25IIII30IIII35IIII40IIII45IIII50
---------------------------------------------------------------------------------------------
 1   -32.3670    -29.4168     1   8
 2   -29.4168    -26.4666     0
 3   -26.4666    -23.5163     0
 4   -23.5163    -20.5661     1   8
 5   -20.5661    -17.6159     1   8
 6   -17.6159    -14.6657     3   8••
 7   -14.6657    -11.7155    13   8••••••••••••
 8   -11.7155     -8.7653    11   8••••••••••
 9    -8.7653     -5.8151    39   8••••••••••••••••••••••••••••••••••••••
10    -5.8151     -2.8649    47   8••••••••••••••••••••••••••••••••••••••••••••••
11    -2.8649      0.0853    45   8••••••••••••••••••••••••••••••••••••••••••••
12     0.0853      3.0355    34   8•••••••••••••••••••••••••••••••••
13     3.0365      5.9857    28   8•••••••••••••••••••••••••••
14     5.9857      8.9359    25   8••••••••••••••••••••••••
15     8.9359     11.8861    17   8••••••••••••••••
16    11.8861     14.8363    17   8••••••••••••••••
17    14.8363     17.7865     9   8••••••••
18    17.7865     20.7366     8   8•••••••
19    20.7366     23.6868     5   8••••
20    23.6868     26.6370     2   8•
---------------------------------------------------------------------------------------------
        SCALING FACTOR = 1
---------------------------------------------------------------------------------------------
```

Average Return = 0.81% per month
Standard Deviation = 9.02% per month
Number of Observations = 306

Exhibit 3 gives the same data for a single security, National Department Stores. Note that the distribution is highly skewed. The arithmetic average return is 0.81 per cent per month over the 306-month period. The most interesting aspect, however, is the standard deviation of month-by-month returns—9.02 per cent per month, more than double that for the diversified portfolio. This result will be discussed further in the next section.

Thus far our discussion of portfolio risk has been confined to a single-period investment horizon such as the next year; that is, the portfolio is held unchanged and evaluated at the end of the year. An obvious question relates to the effect of holding the portfolio for several periods—say for the next 20 years. Will the one-year risks tend to cancel out over time? Given the random-walk nature of security prices, the answer to this question is no. If the risk level (standard deviation) is maintained during each year, the portfolio risk for longer horizons will increase with the horizon length. The standard deviation of possible terminal portfolio values after N years is equal to $\sqrt{N}$ times the standard deviation after one year. Thus the investor cannot rely on the "long run" to reduce his risk of loss.

A final remark should be made before leaving portfolio risk measures. We have implicitly assumed that investors are risk averse, i.e., that they seek to minimize risk for a given level of return. This assumption appears to be valid for most investors in most situations. The entire theory of portfolio selection and capital asset pricing is based on the belief that investors *on the average* are risk averse.

4. Diversification

When one compares the distribution of historical returns for the 100-stock portfolio (Exhibit 2) with the distribution for National Department Stores (Exhibit 3), he discovers a curious relationship. While the standard deviation of returns for the security is double that of the portfolio, its average return is less. Is the market so imperfect that over a long period of time (25 years) it rewarded substantially higher risk with lower average return?

Not so. As we shall now show, not all of the security's risk is relevant. Much of the total risk (standard deviation of return) of National Department Stores was diversifiable. That is, if it had been combined with other securities, a portion of the variation in its returns could have been smoothed out or cancelled by complementary variation in the other securities. The same portfolio diversification effect accounts for the low standard deviation of return for the 100-stock portfolio. In fact, the portfolio standard deviation was less than that of the typical security in the portfolio. Much of the total risk of the component securities had been eliminated by diversification. Since much of the total risk could be eliminated simply by holding a stock in a portfolio, there was no economic requirement for the return earned to be in line with the total risk. Instead, we should expect realized returns to be related to that portion of security risk which cannot be eliminated by portfolio combination.

Diversification results from combining securities having less than perfect correlation (dependence) among their returns in order to reduce portfolio risk. The portfolio return, being simply a weighted average of the individual security returns, is not diminished by diversification. In general, the lower the correlation among security returns, the greater the impact of diversification. This is true regardless of how risky the securities of the portfolio are when considered in isolation.

Ideally, if we could find sufficient securities with uncorrelated returns, we could completely eliminate portfolio risk. This situation is unfortunately not typical in real securities markets where returns are positively correlated to a considerable degree. Thus while portfolio risk can be substantially reduced by diversification, it cannot be entirely eliminated. This can be demonstrated very clearly by measuring the standard deviations of randomly selected portfolios containing various numbers of securities.

In a study of the impact of portfolio diversification on risk, Wagner and Lau [27]* divided a sample of 200 NYSE stocks into six subgroups based on the Standard and Poor's Stock Quality Ratings as of June 1960. The highest quality ratings (A+) formed the first group, the second highest ratings (A) the next group, and so on. Randomly selected portfolios were formed from each of the subgroups containing from 1 to 20 securities. The month-by-month portfolio returns for the 10-year period through May 1970 were then computed for each portfolio (portfolio composition remaining unchanged). The exercise was repeated ten times to reduce the dependence on single samples, and the values for the ten trials were then averaged.

Table 1 shows the average return and standard deviation for portfolios from the first subgroup (A+ quality stocks). The average return is unrelated to the number of issues in the portfolio. On the other hand, the standard deviation of return declines as the number of holdings increases. On the average, approximately 40 per cent of the single security risk is eliminated by forming randomly selected portfolios of 20 stocks

TABLE 1. RISK VERSUS DIVERSIFICATION FOR RANDOMLY SELECTED PORTFOLIOS OF A+ QUALITY SECURITIES

June 1960—May 1970

Number of Securities in Portfolio	Average Return (%/month)	Std. Deviation of Return (%/month)	Correlation with Market	
			R	R^2
1	0.88	7.0	0.54	0.29
2	0.69	5.0	0.63	0.40
3	0.74	4.8	0.75	0.56
4	0.65	4.6	0.77	0.59
5	0.71	4.6	0.79	0.62
10	0.68	4.2	0.85	0.72
15	0.69	4.0	0.88	0.77
20	0.67	3.9	0.89	0.80

Source: Wagner and Lau [27], Table C, p. 53.

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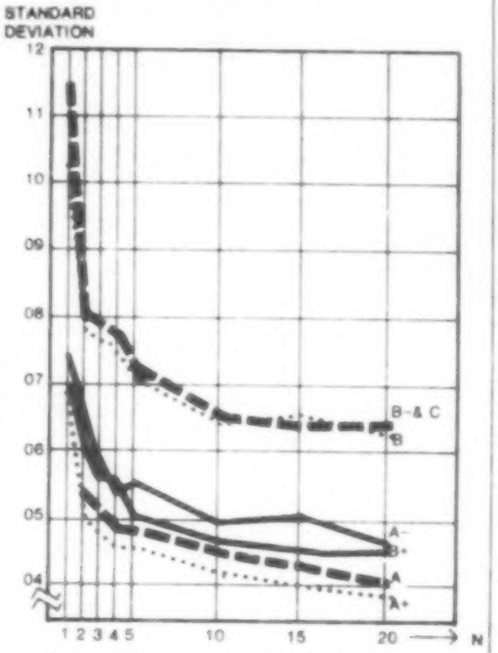
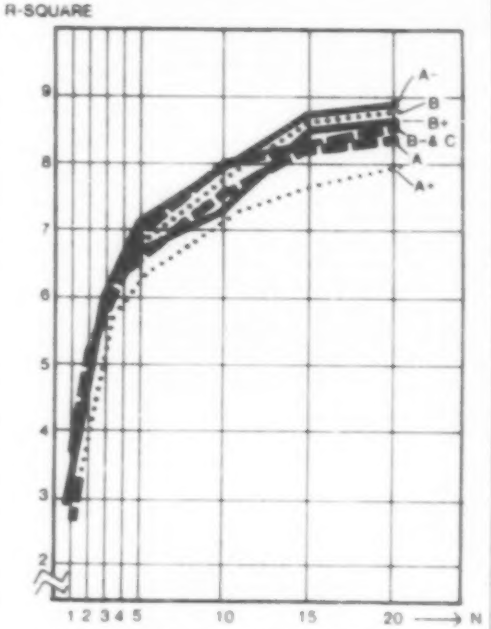
* References appear at end of article.

EXHIBIT 4
STANDARD DEVIATION VERSUS NUMBER OF ISSUES IN PORTFOLIO



Source: Wagner and Lau [27], Exhibit 1, p.50

EXHIBIT 5
CORRELATION VERSUS NUMBER OF ISSUES IN PORTFOLIO



Source: Wagner and Lau [27], Exhibit 2, p.50

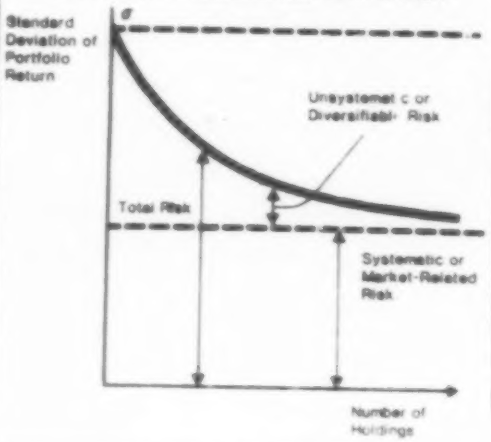
However, it is also evident that additional diversification yields rapidly diminishing reduction in risk. The improvement is slight when the number of securities held is increased beyond, say, 10. Exhibit 4 shows the results for all six quality groups. The figure shows the rapid decline in total portfolio risk as the portfolios are expanded from 1 to 10 securities.

Returning to Table 1, we note from the next to last column in the table that the return on a diversified portfolio follows the market very closely. The degree of association is measured by the correlation coefficient (R) of each portfolio with an unweighted index of all NYSE stocks (perfect positive correlation results in a correlation coefficient of 1.0).[3] The 20-security portfolio has a correlation of 0.89 with the market. The implication is that the risk remaining in the 20-stock portfolio is predominantly a reflection of uncertainty about the performance of the stock market in general. Exhibit 5 shows the results for the six quality groups.

Correlation in Exhibit 5 is represented by the correlation coefficient squared, R^2 (possible values range from 0 to 1.0). The R^2 coefficient has a useful interpretation: it measures the proportion of variability in portfolio return that is attributable to variability in market returns. The remaining variability is risk, which is unique to the portfolio and, as Exhibit 4 shows, can be eliminated by proper diversification of the portfolio. Thus, R^2 measures the degree of portfolio diversification. A poorly diversified portfolio will have a small R^2 (0.30 - 0.40). A well diversified portfolio will have a much higher R^2 (0.85 - 0.95). A perfectly diversified portfolio will have an R^2 of 1.0; that is, all the risk in such a portfolio is a reflection of market risk. Exhibit 5 shows the rapid gain in diversification as the portfolio is expanded from 1 to 2 securities and up to 10 securities. Beyond 10 securities the gains tend to be smaller. Note that increasing the number of issues tends to be less efficient at achieving diversification for the highest quality A+ issues. Apparently the companies comprising this group are more homogeneous than the companies grouped under the other quality codes.

The results show that while some risks can be eliminated via diversification, others cannot. Thus we are led to distinguish between a security's "unsystematic" risk, which can be washed away by mixing the security with other securities in a diversified portfolio, and its "systematic" risk, which cannot be eliminated by diversification. This proposition is illustrated in Exhibit 6. It shows total portfolio risk declining as the number of holdings increases. Increasing diversification gradually tends to eliminate the unsystematic risk, leaving only systematic, i.e., market-related risk. The remaining variability results from the fact that the return on nearly every security depends to some degree on the overall performance of the market. Consequently, the return on a well diversified portfolio is highly correlated with the market, and its variability or uncertainty is basically the uncertainty of the market as a whole. Investors are exposed to market uncertainty no matter how many stocks they hold.

EXHIBIT 6
SYSTEMATIC AND UNSYSTEMATIC RISK



5. The Risk of Individual Securities

In the previous section we concluded that the systematic risk of an individual security is that portion of its total risk (standard deviation of return) which cannot be eliminated by combining it with other securities in a well diversified portfolio. We now need a way of quantifying the systematic risk of a security and relating the systematic risk of a portfolio to that of its component securities. This can be accomplished by dividing security return into two parts: one dependent (i.e., perfectly correlated), and a second independent (i.e., uncorrelated) of market return. The first component of return is usually referred to as "systematic", the second as "unsystematic" return. Thus we have

$$\text{Security Return} = \text{Systematic Return} + \text{Unsystematic Return} \qquad (4)$$

Since the systematic return is perfectly correlated with the market return, it can be expressed as a factor, designated beta (β), times the market return, R_m. The beta factor is a market sensitivity index, indicating how sensitive the security return is to changes in the market level. The unsystematic return, which is independent of market returns, is usually represented by a factor epsilon (ϵ'). Thus the security return, R, may be expressed

$$R = \beta R_m + \epsilon' \qquad (5)$$

For example, if a security had a β factor of 2.0 (e.g., an airline stock), then a 10 per cent market return would generate a systematic return for the stock of 20 per cent. The security return for the period would be the 20 per cent plus the unsystematic component. The unsystematic component depends on factors unique to the company, such as labor difficulties, higher than expected sales, etc.

The security returns model given by Equation (5) is usually written in a way such that the average value of the residual term, ϵ', is zero. This is accomplished by adding a factor, alpha (α), to the model to represent the average value of the unsystematic returns over time. That is, we set $\epsilon' = \alpha + \epsilon$ so that

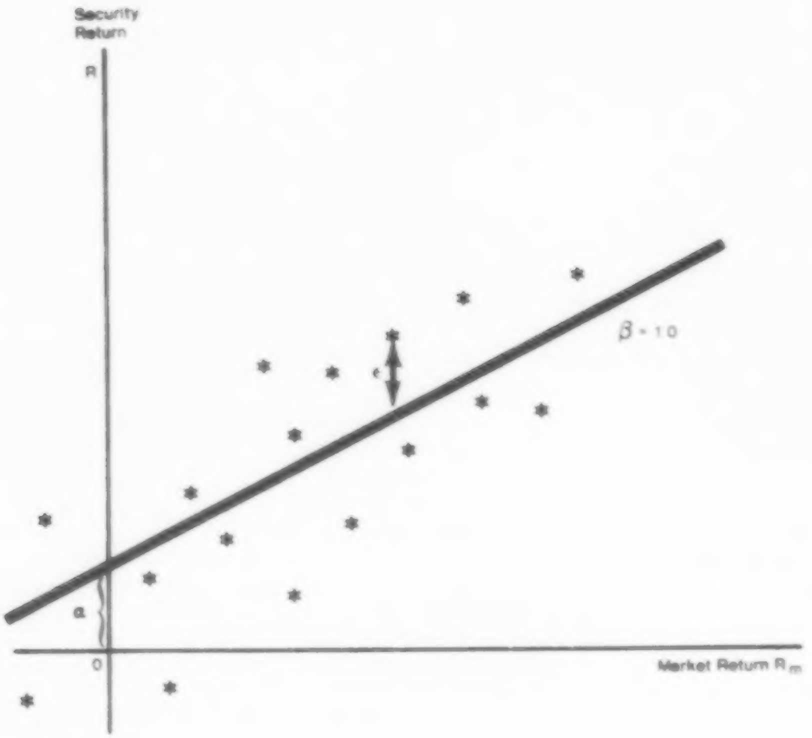
$$R = \alpha + \beta R_m + \epsilon \qquad (6)$$

where the average ϵ over time is equal to zero.

The model for security returns given by Equation (6) is usually referred to as the "market model". Graphically, the model can be depicted as a line fitted to a plot of security returns against rates of return on the market index. This is shown in Exhibit 7 for a hypothetical security.

The beta factor can be thought of as the slope of the line. It gives the expected increase in security return for a one per cent increase in market return. In Exhibit 7, the security has a beta of 1.0. Thus, a ten per cent market return will result, on the average, in a ten per cent security return. The

EXHIBIT 7
THE MARKET MODEL FOR SECURITY RETURNS



Beta (β), the market sensitivity index, is the slope of the line.

Alpha (α), the average of the residual returns, is the intercept of the line on the security axis.

Epsilon (ϵ), the residual returns, are the perpendicular distances of the points from the line.

market-weighted average beta for all stocks is 1.0 by definition.

The alpha factor is represented by the intercept of the line on the vertical security return axis. It is equal to the average value over time of the unsystematic returns (ϵ') on the stock. For most stocks, the alpha factor tends to be small and unstable. (We shall return to alpha later.)

Using the definition of security return given by the market model, the specification of systematic and unsystematic risk is straightforward—they are simply the standard deviations of the two return components.[8]

The systematic risk of a security is equal to β

times the standard deviation of the market return:

$$\text{Systematic Risk} = \beta \sigma_m . \tag{7}$$

The unsystematic risk equals the standard deviation of the residual return factor ϵ:

$$\text{Unsystematic Risk} = \sigma_\epsilon . \tag{8}$$

Given measures of individual security systematic risk, we can now compute the systematic risk of portfolio. It is equal to the beta factor for the portfolio, β_p, times the risk of the market index, σ_m:

$$\text{Portfolio Systematic Risk} = \beta_p \sigma_m . \tag{9}$$

The portfolio beta factor in turn can be shown to be simply an average of the individual security betas, weighted by the proportion of each security in the portfolio, or

$$\beta_p = \sum_{j=1}^{N} X_j \beta_j . \tag{10}$$

where

X_j = the proportion of portfolio market value represented by security j

N = the number of securities.

Thus the systematic risk of the portfolio is simply a weighted average of the systematic risk of the individual securities. If the portfolio is composed of an equal dollar investment in each stock (as was the case for the 100-security portfolio of Exhibit 2), the β_p is simply an unweighted average of the component security betas.

The unsystematic risk of the portfolio is also a function of the unsystematic security risks, but the form is more complex.[7] The important point is that with increasing diversification this risk can be reduced toward zero.

With these results for portfolio risk, it is useful to return to Exhibit 4. The figure shows the decline in portfolio risk with increasing diversification for each of the six quality groups. However, the portfolio standard deviations for each of the six groups are decreasing toward different limits because the average risks (β) of the groups differ.

Table 2 shows a comparison of the standard deviations for the 20-stock portfolios with the predicted lower limits based on average security systematic risks. The lower limit is equal to the average beta for the quality group ($\bar{\beta}$) times the standard deviation of the market return (σ_m). The standard deviations in all cases are close to the predicted values. These results support the contention that portfolio systematic risk equals the average systematic risks of the component securities.

The main results of this section can be summarized as follows: First, as seen from Exhibit 4, roughly 40 to 50 per cent of total security risk can be eliminated by diversification. Second, the remaining systematic risk is equal to the security β times market risk. Third, portfolio systematic risk is a weighted average of security systematic risks.

The implications of these results are substantial. First, we would expect realized rates of return over substantial periods of time to be related to the systematic as opposed to total risk of securities. Since the unsystematic risk is relatively easily eliminated, we should not expect the market to offer a risk premium for bearing it. Second, since

TABLE 2. STANDARD DEVIATIONS OF 20-STOCK PORTFOLIOS AND PREDICTED LOWER LIMITS

June 1960—May 1970

(1) Stock Quality Group	(2) Standard Deviation of 20-Stock Portfolios σ · %/mo	(3) Average Beta Value for Quality Group $\bar{\beta}$	(4) Lower Limit* $\bar{\beta} \cdot \sigma_m$ %/mo
A+	3.94	0.74	3.51
A	4.17	0.80	3.80
A-	4.52	0.89	4.22
B+	4.45	0.87	4.13
B	6.27	1.24	5.89
B- & C	6.32	1.23	5.84

* σ_m = 4.75% per month

Source: Wagner and Lau [27], p. 52, and Table C, p. 53

security systematic risk is equal to the security beta times σ_m (which is common to all securities), beta is useful as a *relative* risk measure. The β gives the systematic risk of a security (or portfolio) relative to the risk of the market index. Thus it is often convenient to speak of systematic risk in relative terms (i.e., in terms of beta rather than beta times σ_m). ■

The remainder of "An Introduction to Risk and Return: Concepts and Evidence" will appear in the May/June issue.

Footnotes

1. The transformation changes nothing of substance since

$$\tilde{M}_1 = (1 + \tilde{R}_P)\, M_0$$
$$= M_0 + M_0 \tilde{R}_P \ ,$$

where

$\tilde{M}_1$ = terminal portfolio value

$\tilde{R}_P$ = portfolio return.

Since $\tilde{M}_1$ is a linear function of $\tilde{R}_P$, any risk measures developed for the portfolio return will apply equally to the terminal market value.

2. Risk measures based on below-the-average variation are analytically difficult to deal with. H. Markowitz, in Chapter 9 of [18], develops a semivariance statistic which measures variability below the mean and compares it with the more commonly used variance calculation.
3. See for example M. E. Blume [2].
4. This result can be illustrated as follows. The portfolio market value after N years, $\tilde{M}_N$, is equal to

$$\tilde{M}_N = M_0 \left[(1 + \tilde{R}_{p1})(1 + \tilde{R}_{p2}) \ldots (1 + \tilde{R}_{pN})\right] ,$$

where M_0 is the initial value, and $\tilde{R}_{pt}$ (t = 1, . . ., N) is the return during year t [as given by Equation (1a)]. For reasonably small values of the annual returns, the above expression can be approximated by

$$\tilde{M}_N = M_0 \left[1 + \tilde{R}_{p1} + \tilde{R}_{p2} + \ \ldots \ + \tilde{R}_{pN}\right] .$$

Now, if the annual returns, $\tilde{R}_{pt}$ are independently and identically distributed with variance σ^2, the variance of $\tilde{M}_N$ will equal $(M_0)^2 N\sigma^2$, or N times the variance after one year. Therefore, the standard deviation of the terminal value will equal $\sqrt{N}$ times the standard deviation after one year. The key assumption of independence of portfolio returns over time is realistic, since security returns appear to follow a random walk through time.

A similar result could be obtained without the restriction on the size of the $\tilde{R}_{pt}$ if we had dealt with continuously, as opposed to annually, compounded rates of return. However, the analysis would be more complicated.

5. Two securities with perfectly correlated return patterns will have a correlation coefficient of 1.0. Conversely, if the return patterns are perfectly negative correlated, the correlation coefficient will equal -1. Two securities with uncorrelated (i.e., statistically unrelated returns will have a correlation coefficient of zero. The average correlation coefficient between returns for NYSE securities and the S&P 500 Stock Index during the 1945-1970 period was approximately 0.5.
6. The relationship between the risk components is given by

$$\sigma^2 = \beta^2 \sigma_m^2 + \ \sigma_{\epsilon}^2 \ .$$

This follows directly from Equation (5) and the assumption of statistical independence of R_m and ϵ'. The R^2 term previously discussed is the ratio of systematic to total risk (both measured in terms of variance):

$$R^2 = \frac{\beta^2 \sigma_m^2}{\sigma^2} \ .$$

Note also that the R^2 is the square of the correlation coefficient between security and market returns.

7. Assuming the unsystematic returns (ϵ'_j) of securities to be uncorrelated (reasonably true in practice), the unsystematic portfolio risk is given by

$$\sigma^2(\epsilon'_p) = \sum_{j=1}^{N} X_j^2 \sigma^2(\epsilon'_j) \ ,$$

where $\sigma^2(\epsilon'_j)$ is the unsystematic risk for stock j. Assume the portfolio is made up of equal investment in each security and $\bar{\sigma}^2(\epsilon')$ is the average value of the $\sigma^2(\epsilon'_j)$. Then, X_j = 1/N and

$$\sigma^2(\epsilon'_p) = \frac{1}{N} \bar{\sigma}^2(\epsilon') \ ,$$

which (assuming $\bar{\sigma}^2(\epsilon')$ is finite) obviously approaches zero as the number of issues in the portfolio increases.

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by Franco Modigliani and Gerald A. Pogue

An Introduction to Risk and Return

Concepts and Evidence - II

6. The Relationship Between Expected Return and Risk: The Capital Asset Pricing Model*

The first part of this article developed two measures of risk: one is a measure of total risk (standard deviation), the other a relative index of systematic or nondiversifiable risk (beta). The beta measure would appear to be the more relevant for the pricing of securities. Returns expected by investors should logically be related to systematic as opposed to total risk. Securities with higher systematic risk should have higher expected returns.[1]

The question to be considered now is the form of the relationship between risk and return. In this section we describe a relationship called the "Capital Asset Pricing Model" (CAPM), which is based on elementary logic and simple economic principles. The basic postulate underlying finance theory is that assets with the same risk should have the same expected rate of return. That is, the prices of assets in the capital markets should adjust until equivalent risk assets have identical expected returns.

To see the implications of this postulate, let us consider an investor who holds a risky portfolio[2] with the same risk as the market portfolio (beta equal to 1.0). What return should he expect? Logically, he should expect the same return as that of the market portfolio.

Let us consider another investor who holds a riskless portfolio (beta equal to zero). The investor in this case should expect to earn the rate of return on riskless assets such as treasury bills. By taking no risk, he earns the riskless rate of return.

Now let us consider the case of an investor who holds a mixture of these two portfolios. Assuming he invests a proportion X of his money in the risky portfolio and (1 − X) in the riskless portfolio, what risk does he bear and what return should he expect? The risk of the composite portfolio is easily computed when we recall that the beta of a portfolio is simply a weighted average of the component security betas, where the weights are the portfolio proportions. Thus the portfolio beta, β_p, is a weighted average of the beta of the market portfolio and the beta of the risk-free rate. However, the market beta is 1.0, and that of the risk-free rate is zero. Therefore

$$\begin{aligned}\beta_p &= (1-X)\cdot 0 + X\cdot 1,\\ &= X.\end{aligned}\tag{11}$$

Thus β_p is equal to the fraction of his money invested in the risky portfolio. If 100 per cent or less of the investor's funds is invested in the risky portfolio, his portfolio beta will be between zero and 1.0. If he borrows at the risk-free rate and invests the proceeds in the risky portfolio, his portfolio beta will be greater than 1.0.

The expected return of the composite portfolio is also a weighted average of the expected returns on the two-component portfolios; that is,

$$E(R_p) = (1-X)\cdot R_f + X\cdot E(R_m),\tag{12}$$

where $E(R_p)$, $E(R_m)$, and R_f are the expected returns on the portfolio, the market index, and the risk-free rate. Now, from Equation (11) we know

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1. Footnotes appear at end of article.

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* *This article is the second part of a two-part article which was originally prepared as Chapter 2 of* A Study of Investment Performance Fees *(Heath-Lexington Books, forthcoming 1974). The research was supported by a grant from the Investment Company Institute, Washington, D.C. The article will also appear in the forthcoming* Financial Analysts Handbook *(Sumner Levine, editor; to be published by Dow Jones-Irwin). Franco Modigliani is Professor of Finance, Sloan School of Management, M.I.T. Gerald Pogue is Professor of Finance at Baruch College, City University of New York.*

that X is equal to β_p. Substituting into Equation (12), we have

$$E(R_p) = (1-\beta_p)\cdot R_F + \beta_p \cdot E(R_m),$$

or

$$E(R_p) = R_F + \beta_p \cdot \left(E(R_m) - R_F\right). \qquad (13)$$

Equation (13) is the Capital Asset Pricing Model (CAPM), an extremely important theoretical result. It says that the expected return on a portfolio should exceed the riskless rate of return by an amount which is proportional to the portfolio beta. That is, the relationship between return and risk should be linear.

The model is often stated in "risk-premium" form. Risk premiums are obtained by subtracting the risk-free rate from the rates of return. The expected portfolio and market risk premiums (designated $E(r_p)$ and $E(r_m)$ respectively) are given by

$$E(r_p) = E(R_p) - R_F, \qquad (14a)$$

$$E(r_m) = E(R_M) - R_F \qquad (14b)$$

Substituting these risk premiums into Equation (13), we obtain

$$E(r_p) = \beta_P \cdot E(r_m). \qquad (15)$$

In this form, the CAPM states that the expected risk premium for the investor's portfolio is equal to its beta value times the expected market risk premium.

We can illustrate the model by assuming that short-term (risk-free) interest rate is 6 per cent and the expected return on the market is 10 per cent. The expected risk premium for holding the market portfolio is just the difference between the 10 per cent and the short-term interest rate of 6 per cent, or 4 per cent. Investors who hold the market portfolio expect to earn 10 per cent, which is 4 per cent greater than they could earn on a short-term market instrument for certain. In order to satisfy Equation (13), the expected return on securities or portfolios with different levels of risk must be:

Expected Return for Different Levels of Portfolio Beta

Beta	Expected Return
0.0	6%
0.5	8%
1.0	10%
1.5	12%
2.0	14%

The predictions of the model are inherently sensible. For safe investments ($\beta = 0$), the model predicts that investors would expect to earn the risk-free rate of interest. For a risky investment ($\beta > 0$) investors would expect a rate of return proportional to the market sensitivity (β) of the investment. Thus, stocks with lower than average market sensitivities (such as most utilities) would offer expected returns less than the expected market return. Stocks with above average values of beta (such as most airline securities) would offer expected returns in excess of the market.

In our development of CAPM we have made a number of assumptions that are required if the model is to be established on a rigorous basis. These assumptions involve investor behavior and conditions in the capital markets. The following is a set of assumptions that will allow a simple derivation of the model.

(a) The market is composed of risk-averse investors who measure risk in terms of standard deviation of portfolio return. This assumption provides a basis for the use of beta-type risk measures.

(b) All investors have a common time horizon for investment decision making (e.g., one month, one year, etc.). This assumption allows us to measure investor expectations over some common interval, thus making comparisons meaningful.

(c) All investors are assumed to have the same expectations about future security returns and risks. Without this assumption, the analysis would become much more complicated.

(d) Capital markets are perfect in the sense that all assets are completely divisible, there are no transactions costs or differential taxes, and borrowing and lending rates are equal to each other and the same for all investors. Without these conditions frictional barriers would exist to the equilibrium conditions on which the model is based.

While these assumptions are sufficient to derive the model, it is not clear that all are necessary in their current form. It may well be that several of the assumptions can be substantially relaxed without major change in the form of the model. A good deal of research is currently being conducted toward this end.

While the CAPM is indeed simple and elegant, these qualities do not in themselves guarantee that it will be useful in explaining observed risk-return patterns. In Section 8 we will review the empirical literature on attempts to verify the model.

7. Measurement of Security and Portfolio Beta Values

The basic data for estimating betas are past rates of return earned over a series of relatively short intervals—usually days, weeks, or months. For example, in Tables 3 and 4 we present calculations

TABLE 3: REGRESSION STATISTICS FOR 30 RANDOMLY SELECTED SECURITIES*
January 1945 - June 1970

SECURITY	(1) NOBS	(2) ALPH	(3) SE.A	(4) BETA	(5) SE.B	(6) SE.R	(7) R**2	(8) ARPJ	(9) SD.R	(10) CRPJ
1 City Investing Co.	306.00	0.30	0.53	1.67	0.14	9.20	31.43	1.45	11.09	0.87
2 Foster Wheeler	306.00	-0.12	0.49	1.57	0.13	8.36	32.98	0.96	10.20	0.46
3 Pennsylvania Dixie	306.00	-0.20	0.47	1.40	0.12	8.15	29.33	0.77	9.67	0.33
4 National Gypsum Co.	306.00	-0.18	0.32	1.38	0.08	5.45	47.29	0.77	7.49	0.50
5 Radio Corp. Of America	306.00	0.02	0.38	1.35	0.10	6.60	37.02	0.95	8.30	0.62
6 Fox Film Corp.	306.00	-0.04	0.53	1.31	0.14	9.15	22.35	0.87	10.36	0.38
7 Intercontinental Rubber	306.00	0.69	0.64	1.28	0.17	10.95	16.13	1.58	11.94	0.92
8 National Department	306.00	-0.05	0.45	1.26	0.12	7.73	27.05	0.81	9.04	0.41
9 Phillips Jones Corp.	306.00	0.36	0.44	1.25	0.12	7.54	27.89	1.22	8.86	0.85
10 Chrysler Corp.	306.00	-0.26	0.37	1.21	0.10	6.29	34.12	0.58	7.73	0.28
11 American Hide & Leather	306.00	0.55	0.66	1.16	0.17	11.36	12.78	1.35	12.14	0.67
12 Adams Express	306.00	0.11	0.23	1.16	0.06	3.93	54.87	0.91	5.84	0.75
13 Caterpillar Tractor	306.00	0.43	0.32	1.14	0.08	5.45	38.09	1.22	6.92	0.99
14 Continental Steel Co.	306.00	0.21	0.36	1.12	0.10	6.22	31.31	0.99	7.50	0.72
15 Marland Oil Co.	306.00	0.06	0.29	1.11	0.08	4.99	40.69	0.82	6.47	0.62
16 Air Reduction Co.	306.00	-0.59	0.29	1.08	0.08	4.98	39.73	0.16	6.41	-0.05
17 National Aviation	306.00	0.22	0.39	1.04	0.10	6.71	25.15	0.94	7.74	0.65
18 NA Tomas Co.	306.00	0.28	0.63	1.01	0.17	10.88	10.72	0.98	11.50	0.37
19 NYSE Index	306.00	0.0	0.0	1.00	0.0	0.0	0.0	0.69	3.73	0.62
20 American Ship Building	306.00	0.31	0.52	0.99	0.14	9.01	14.53	0.99	9.73	0.54
21 James Talcott	306.00	0.33	0.42	0.99	0.11	7.23	20.43	1.01	8.09	0.68
22 Jewel Tea Co. Inc.	306.00	0.21	0.32	0.95	0.08	5.42	30.14	0.67	6.47	0.66
23 International Carrier	306.00	0.34	0.26	0.93	0.07	4.39	38.41	0.98	5.58	0.83
24 Keystone Steel & Wire	306.00	0.18	0.30	0.84	0.08	5.19	26.90	0.76	6.05	0.58
25 Swift & Co.	306.00	-0.09	0.30	0.81	0.08	5.08	26.08	0.47	5.89	0.30
26 Southern California	306.00	0.00	0.22	0.77	0.06	3.77	36.60	0.53	4.72	0.42
27 Bayuk Cigars	306.00	-0.04	0.39	0.71	0.10	6.76	13.49	0.45	7.26	0.19
28 First National Store	306.00	-0.08	0.31	0.67	0.08	5.33	18.01	0.38	5.88	0.21
29 National Linen Service	306.00	0.61	0.33	0.63	0.09	5.75	14.50	1.04	6.20	0.86
30 American Snuff	306.00	0.17	0.25	0.54	0.07	4.33	17.74	0.54	4.77	0.43
31 Homestake Mining Co.	306.00	0.16	0.38	0.24	0.10	6.60	1.77	0.33	6.65	0.11
32 Commercial Paper	306.00	0.0	0.0	0.0	0.0	0.0	0.0	0.28	0.17	0.28
...Mean Sec. Values	306.00	0.13	0.36	1.05	0.10	6.76	27.25	0.86	7.88	0.54
...Standard Deviations	0.0	0.28	0.12	0.31	0.03	2.10	11.85	0.33	2.13	0.26

*Based on monthly data, regression results sorted by beta (column 4).

based on month-by-month rates of return for the periods January 1945 to June 1970 (security betas) and January 1960 to December 1971 (mutual fund betas). The returns were calculated using Equation (1a).

It is customary to convert the observed rates of return to risk premiums. Section 6 showed that risk premiums are obtained by subtracting the rates of return that could have been achieved by investing in short-maturity risk-free assets, such as treasury bills or prime commercial paper. This removes a source of "noise" from the data. The noise stems from the fact that observed returns may be higher in some years simply because risk-free rates of interest are higher. Thus, an observed rate of return of eight per cent might be regarded as satisfactory if it occurred in 1960, but as a relatively low rate of return when interest rates were at all-time highs

DESCRIPTION OF COLUMNS IN TABLES 3 AND 4

Column Number	Symbol	Description
1	NOBS	Number of Monthly Returns
2	ALPHA	The Estimated Alpha Value
3	SE-A	Standard Error of Alpha
4	BETA	Estimated Beta Coefficient
5	SE-B	Standard Error of Beta
6	SE-R	Standard Error of the Regression—an Estimate of the Unsystematic Risk
7	R**2	R^2 Expressed in Percentage Terms
8	ARPJ	Arithmetic Average of Monthly Risk Premiums
9	SD-R	Standard Deviation of Monthly Risk Premiums
10	CRPJ	Geometric (Time-Weighted) Average of Monthly Risk Premiums

TABLE 4. REGRESSION STATISTICS FOR 49 MUTUAL FUNDS*
January 1960 - December 1971

SECURITY	(1) NOBS	(2) ALPH	(3) SE.A	(4) BETA	(5) SE.B	(6) SE.R	(7) R**2	(8) ARPJ	(9) SD.R	(10) CRPJ
1 McDonnell Fund	144.00	0.58	0.82	1.50	0.22	9.76	25.18	1.13	11.24	0.67
2 Value Line Spec. Sit.	144.00	0.02	0.40	1.48	0.11	4.78	57.62	0.57	7.32	0.30
3 Keystone S-4	144.00	0.03	0.28	1.43	0.08	3.38	71.77	0.55	6.34	0.35
4 Chase Fund of Boston	144.00	0.11	0.33	1.42	0.09	3.94	64.78	0.63	6.61	0.41
5 Equity Progress	144.00	-0.54	0.41	1.26	0.11	4.85	48.89	-0.08	6.77	-0.31
6 Oppenheimer Fund	144.00	0.42	0.24	1.23	0.08	2.89	72.16	0.88	5.46	0.73
7 Fidelity Trend Fund	144.00	0.79	0.29	1.23	0.08	3.52	63.39	1.24	5.80	1.07
8 Fidelity Capital	144.00	0.41	0.24	1.20	0.06	2.81	72.17	0.85	5.31	0.71
9 Keystone K-2	144.00	0.08	0.22	1.17	0.06	2.63	73.90	0.51	5.13	0.38
10 Delaware Fund	144.00	0.18	0.19	1.15	0.05	2.32	77.62	0.60	4.90	0.48
11 Keystone S-3	144.00	0.18	0.19	1.14	0.05	2.32	77.50	0.60	4.88	0.48
12 Putnam Growth Fund	144.00	0.21	0.19	1.13	0.05	2.25	78.19	0.62	4.80	0.51
13 Scudder Special Fund	144.00	0.39	0.28	1.12	0.07	3.33	61.93	0.80	5.37	0.66
14 Energy Fund	144.00	0.06	0.18	1.10	0.05	2.18	78.39	0.46	4.67	0.35
15 One William Street	144.00	0.13	0.22	1.06	0.06	2.66	69.33	0.52	4.78	0.41
16 The Dreyfus Fund	144.00	0.17	0.14	1.04	0.04	1.69	84.40	0.55	4.26	0.46
17 Mass. Investors Gr. Stk.	144.00	0.15	0.16	1.03	0.04	1.98	79.65	0.52	4.34	0.43
18 Windsor Fund	144.00	0.18	0.16	1.03	0.04	1.95	79.87	0.56	4.33	0.47
19 Axe-Houghton Stock	144.00	0.39	0.30	1.02	0.08	3.62	52.96	0.76	5.26	0.62
20 S&P 500 Stock Index	144.00	0.0	0.0	1.00	0.0	0.0	0.0	0.37	3.76	0.30
21 T. Rowe Price Gr. Stk.	144.00	0.05	0.14	0.98	0.04	1.72	82.08	0.41	4.06	0.32
22 Mass. Investors Trust	144.00	-0.02	0.14	0.97	0.04	1.72	82.07	0.34	4.04	0.26
23 Bullock Fund	144.00	0.09	0.19	0.96	0.05	2.32	71.10	0.44	4.29	0.35
24 Keystone S-2	144.00	0.04	0.12	0.96	0.03	1.45	86.12	0.39	3.89	0.31
25 Eaton & Howard Stock	144.00	-0.05	0.13	0.95	0.03	1.52	84.75	0.30	3.89	0.23
26 The Colonial Fund	144.00	0.06	0.19	0.95	0.05	2.27	71.24	0.41	4.23	0.32
27 Fidelity Fund	144.00	0.15	0.11	0.95	0.03	1.31	88.08	0.50	3.79	0.43
28 Invest. Co. of America	144.00	0.26	0.20	0.95	0.05	2.40	68.79	0.61	4.29	0.51
29 Hamilton Funds - HDA	144.00	-0.12	0.23	0.93	0.06	2.73	62.55	0.22	4.44	0.12
30 Affiliated Fund	144.00	0.08	0.10	0.90	0.03	1.22	88.55	0.41	3.59	0.34
31 Keystone S-1	144.00	0.03	0.10	0.88	0.03	1.21	88.18	0.35	3.51	0.29
32 Axe-Houghton Fund B	144.00	0.01	0.20	0.86	0.05	[illegible]	63.68	0.32	4.03	0.24
33 American Mutual Fund	144.00	0.20	0.20	0.85	0.05	2.38	64.35	0.51	3.97	0.43
34 Pioneer Fund	144.00	0.24	0.16	0.84	0.04	1.88	73.85	0.55	3.67	0.48
35 Chemical Fund	144.00	0.57	0.25	0.83	0.07	3.03	51.50	0.88	4.33	0.79
36 Stein R&F Balanced Fd.	144.00	0.06	0.10	0.79	0.03	1.21	86.05	0.35	3.22	0.30
37 Puritan Fund	144.00	0.19	0.15	0.78	0.04	1.79	72.89	0.48	3.43	0.42
38 Value Line Income Fd.	144.00	0.07	0.17	0.78	0.04	2.01	67.96	0.36	3.54	0.29
39 Geo. Putnam Fd. Boston	144.00	0.07	0.10	0.77	0.03	1.18	85.75	0.35	3.12	0.30
40 Anchor Income	144.00	-0.03	0.13	0.74	0.04	1.60	75.24	0.24	3.21	0.19
41 Loomis-Sayles Mutual	144.00	0.05	0.10	0.74	0.03	1.22	83.96	0.32	3.04	0.27
42 Wellington Fund	144.00	-0.12	0.13	0.72	0.03	1.54	75.60	0.14	3.11	0.09
43 Massachusetts Fund	144.00	0.04	0.11	0.72	0.03	1.26	82.16	0.30	2.98	0.26
44 Nation-Wide Sec.	144.00	-0.32	0.15	0.67	0.04	1.78	66.45	-0.08	3.07	-0.12
45 Eaton & Howard Bal. Fd.	144.00	-0.07	0.12	0.62	0.03	1.46	71.62	0.16	2.74	0.12
46 American Business Shares	144.00	0.12	0.09	0.53	0.02	1.10	76.96	0.31	2.28	0.29
47 Keystone K-1	144.00	0.01	0.11	0.53	0.03	1.32	69.59	0.21	2.39	0.18
48 Keystone B-4	144.00	0.12	0.13	0.30	0.03	1.51	35.82	0.23	1.88	0.21
49 Keystone B-2	144.00	0.05	0.10	0.16	0.03	1.16	22.03	0.11	1.31	0.10
50 Keystone B-1	144.00	-0.08	0.10	0.07	0.03	1.21	4.43	-0.06	1.23	-0.07
51 30 Day Treasury Bills	144.00	0.0	0.0	0.0	0.0	0.0	0.0	0.34	0.12	0.34
... Mean Sec. Values	144.00	0.12	0.19	0.93	0.05	2.32	69.25	0.46	4.25	0.36
... Standard Deviations	0.0	0.22	0.12	0.30	0.03	1.42	17.50	0.27	1.64	0.23

*Based on monthly data, regression results sorted by beta (column 4).

in 1969. Rates of return expressed as risk premiums will be denoted by small r's.

The market model of Equation (6), when expressed in risk-premium form, is the basic equation used to estimate beta. The market model in risk-premium form is given by

$$r = \alpha + \beta r_m + \epsilon . \tag{16a}$$

The use of risk premiums instead of returns as in Equation (6) simply changes the interpretation of alpha, leaving beta unchanged. In the return form, the expected value of alpha as given by the CAPM is $R_f(1 = B_p)$ [compare Equations (6) and (13)]. In the risk-premium form, the expected value of alpha is zero [compare Equations (15) and (16a)]. In the latter case, measured values of alpha different from zero can thus be interpreted as an *excess return* earned by a stock or portfolio beyond the return predicted by the CAPM on the basis of the asset's beta value. (More on the interpretation of alpha in Section 9).

Beta for a security is calculated by regressing the observed security risk premiums, r, on the observed risk premiums on the market, r_m. By this procedure we are, in effect, estimating the parameters of the market model of Equation (16a). The equation of the fitted line is

$$r = \hat{\alpha} + \hat{\beta} r_m + \hat{\epsilon} , \tag{16b}$$

where $\hat{\alpha}$ is the intercept of the fitted line and $\hat{\beta}$ represents the stock's systematic risk. The $\hat{\epsilon}$ term represents variation about the line resulting from the unsystematic component of return. We have put hats (^) over the α, β and ϵ terms to indicate that these are estimated values. It is important to remember that these estimated values may differ substantially from the true values because of statistical measurement difficulties. However, the extent of possible error can be measured, and we can indicate a range within which the true value is almost certain to lie.

Exhibit 8 shows a risk-premium plot and fitted line for National Department Stores. The market is represented by a weighted index of all NYSE securities. The plot is based on monthly data during the period January 1945 to June 1970.

The estimated beta is 1.26, indicating above average systematic risk. The estimated alpha is −0.05 per cent per month, indicating that the excess return on the security averaged −0.60 per cent per year over the 25-year period. The correlation coefficient is 0.52; thus 27 per cent of the variance of security returns resulted from market movements. The remainder was due to factors unique to the company.

Our interpretation of the estimated alpha and beta values must be conditioned by the degree of possible measurement error. The measurement error is estimated by "standard error" coefficients associated with alpha and beta.

For example, the standard error of beta is 0.12. Thus, the probability is about 66 per cent that the true beta will lie between 1.26 ± 0.12, and about 95 per cent that it will lie between 1.26 ± 0.24 (i.e., plus or minus two times the standard error). Thus we can say with high confidence that National Department Stores has above average risk (i.e., true beta greater than 1.0).

The standard error for alpha is 0.45, which is large compared with the estimated value of −0.05. Thus we cannot conclude that the true alpha is different from zero, since zero lies well within the range of estimated alpha plus or minus one standard error (i.e., −0.05 ± 0.45).

Table 4 presents the same type of regression results for a random collection of 30 NYSE stocks.[3] The table contains the following items: Column (1) gives the number of monthly observations, columns (2) and (3) the estimated alpha ($\hat{\alpha}$) and its standard error, columns (4) and (5) the estimated beta ($\hat{\beta}$) and its standard error, column (6) the unsystematic risk $\hat{\sigma}_\epsilon$, column (7) the R^2 in percentage terms, columns (8) and (9) the arithmetic average of monthly risk premiums and the standard deviation, and column (10) the geometric mean risk premium. The results are ranked in terms of descending values of estimated beta. The table includes summary results for the NYSE market index and the prime commercial paper risk-free rate.[4] The last two rows of the table give average values and standard deviations for the sample. The average beta, for example, is 1.05, slightly higher than the average of all NYSE stocks.

The beta value for a portfolio can be estimated in two ways. One method is to compute the beta of all portfolio holdings and weight the results by portfolio representation. However, this method has the disadvantage of requiring beta calculations for each individual portfolio asset. The second method is to use the same computation procedures used for stocks, but to apply them to the portfolio returns. In this way we can obtain estimates of portfolio betas without explicit consideration of the portfolio securities. We have used this approach to compute portfolio and mutual fund beta values.

Exhibit 9 shows the plot of the monthly risk premiums on the 100-stock portfolio against the NYSE index for the same 1945-1970 period. As in the case of National Department Stores, the best-fit line has been put through the points using regression analysis. The slope of the line ($\hat{\beta}$) is equal to 1.10, with a standard error of 0.03. Note the substantial reduction in the standard error term compared to the security examples. The estimated alpha is 0.14, with a standard error of 0.10. Again, we cannot conclude that the true alpha is different from zero. Note that the points group much closer to the line than in the National Department Store plot. This results, of course, from the fact that much of the unsystematic risk causing the points to be scattered around the regression line in Exhibit 8 has been climated. The reduction is evidenced by the R^2 measure of 0.87 (versus 0.27 for National Department Stores). Thus the market explains more than three times as much of the return variation of the portfolio than for the stock.

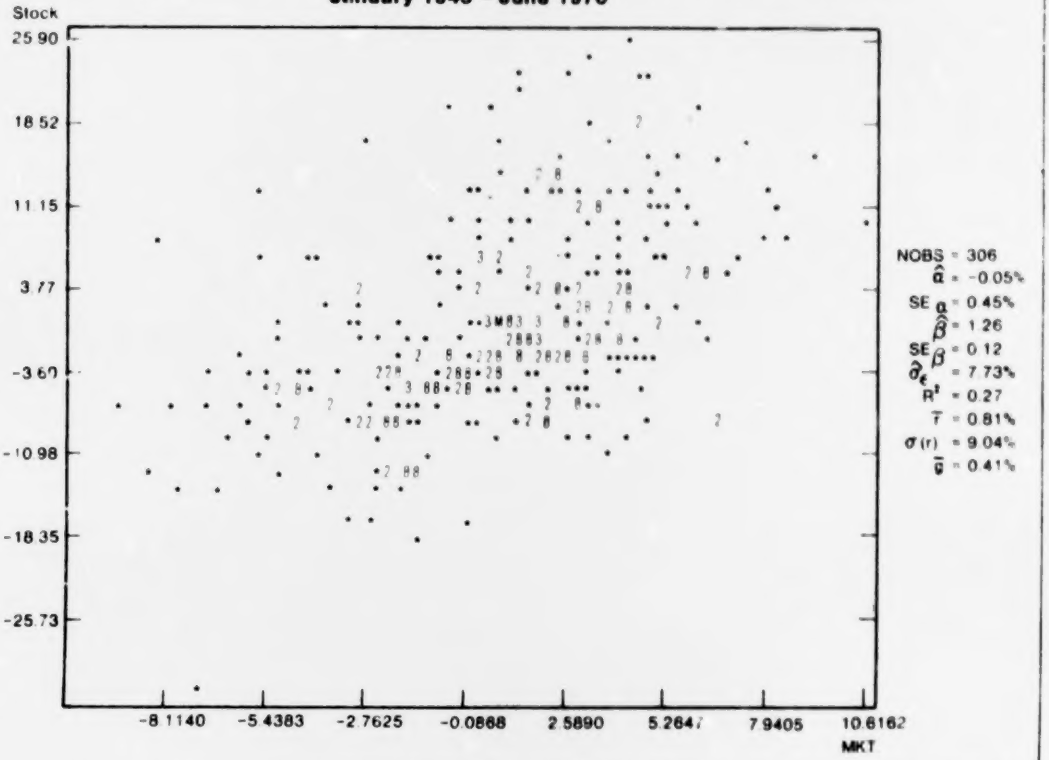
Table 5 gives regression results for a sample of 49 mutual funds. The calculations are based on monthly risk premiums for the period January 1960 to December 1971. The market is represented by the Standard & Poor's 500 Stock Index. Average values and standard deviations for the 49 funds in the sample are shown in the last two rows of the table. The average beta value for the group is 0.93—indicating, on the average, that the funds were less risky than the market index. Note the relatively low beta values of the balanced and bond funds, in particular the Keystone B1, B2, and B4 bond funds. This result is due to the low systematic risk of the bond portfolios.

Up to this point we have shown that it is a relatively easy matter to estimate beta values for stocks, portfolios, and mutual funds. Now, if the

Exhibit 8

RETURNS ON NATIONAL DEPARTMENT STORES VS. NYSE INDEX (% PER MONTH)

January 1945 - June 1970



beta values are to be useful for investment decision making, they must be predictable. Beta values based on historical data should provide considerable information about future beta values if past measures are to be useful. How predictable are the betas estimated for stocks, portfolios of stocks, and mutual funds? Fortunately, we have empirical evidence at each level.

Robert A. Levy [13]* has conducted tests of the short-run predictability (also referred to as stationarity) of beta coefficients for securities and unmanaged portfolios of securities. Levy's results are based on weekly returns for 500 NYSE stocks for the period December 30, 1960 through December 18, 1970 (520 weeks). Betas were developed for each security for ten non-overlapping 52-week periods. To measure stationarity, Levy correlated the 500 security betas from each 52-week period (the historical betas) with the 52-week betas in the following period (the future betas). Thus nine correlation studies were performed for the ten periods.

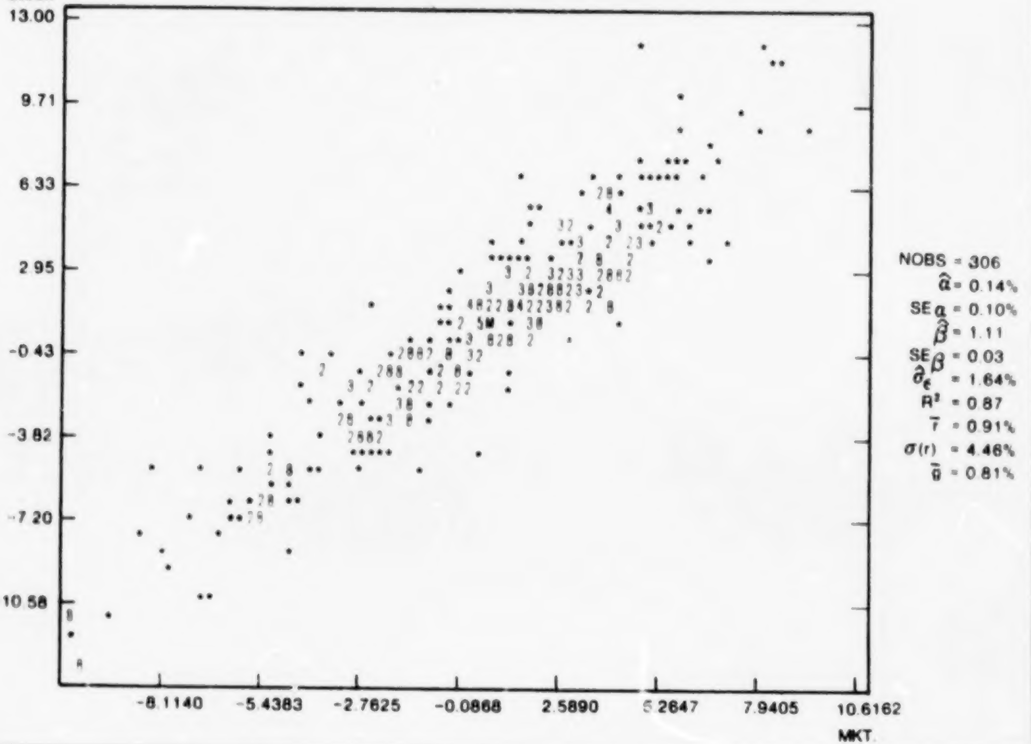
To compare the stationarity of security and portfolio betas, Levy constructed portfolios of 5, 10, 25, and 50 securities and repeated the same correlation analysis for the historical portfolio betas and future beta values for the same portfolios in the subsequent period. The portfolios were constructed by ranking security betas in each period and partitioning the list into portfolios containing 5, 10, 25 and 50 securities. Each portfolio contained an equal investment in each security.

The results of Levy's 52-week correlation studies are presented in Table 5. The average values of the correlation coefficients from the nine trials were 0.486, 0.769, 0.853, 0.939, and 0.972 for portfolios of 1, 5, 10, 25, and 50 stocks, respectively.

Exhibit 9

RETURNS ON 100 STOCK PORTFOLIO VERSUS NYSE INDEX (% PER MONTH)

January 1946 – June 1970



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* References appear at end of article.

Correspondingly, the average percentages of the variation in future betas explained by the historical betas are 23.6, 59.1, 72.8, 88.2, and 94.5.

The results show the beta coefficients to be very predictable for large portfolios and progressively less predictable for smaller portfolios and individual securities. These conclusions are not affected by changes in market performance. Of the nine correlation studies, five covered forecast periods during which the market performance was the reverse of the preceding period (61-62, 62-63, 65-66, 66-67, and 68-69). Notably, the betas were approximately as predictable over these five reversal periods as over the remaining four intervals.[5]

The question of the stability of mutual fund beta values is more complicated. Even if, as seen above, the betas of large unmanaged portfolios are very predictable, there is no a priori need for mutual fund betas to be comparatively stable. Indeed, the betas of mutual fund portfolios may change substantially over time by design. For example, a portfolio manager may tend to reduce the risk exposure of his fund prior to an expected market decline and raise it prior to an expected market upswing. However, the range of possible values for beta will tend to be restricted, at least in the longer run, by the fund's investment objective. Thus while one does not expect the same standard of predictability as for large unmanaged portfolios, it may nevertheless be interesting to examine the extent to which fund betas are predictable.

Pogue and Conway [21] have conducted tests for a sample of 90 mutual funds. The beta values for the period January 1969 through May 1970 were correlated with values from the subsequent period from June 1970 through October 1971. To test the sensitivity of the results to changes in the return measurement interval, the betas for each sub-period were measured for daily, weekly, and monthly returns. The betas were thus based on very different numbers of observations, namely 357, 74, and 17, respectively. The resulting correlation coefficients were 0.915, 0.895, and 0.703 for daily, weekly, and monthly betas. Correspondingly, the average percentages of variation in second-period betas explained by first-period values are 84, 81, and 49, respectively. The results support the contention that historical betas contain useful information about future values. However, the degree of predictability depends on the extent to which measurement errors have been eliminated from beta estimates. In the Pogue-Conway study, the shift from monthly to daily returns reduced the average standard error of the estimated beta values from 0.11 to 0.03, a 75 per cent reduction. The more accurate daily estimates resulted in a much higher degree of beta predictability, the correlation between sub-period betas increasing from 0.703 to 0.915.[6]

Exhibit 10 shows a Pogue-Conway plot of the first-period versus second-period betas based on daily returns. The figure illustrates the high degree of correlation between first- and second-period betas.

In summary, we can conclude that estimated individual security betas are not highly predictable

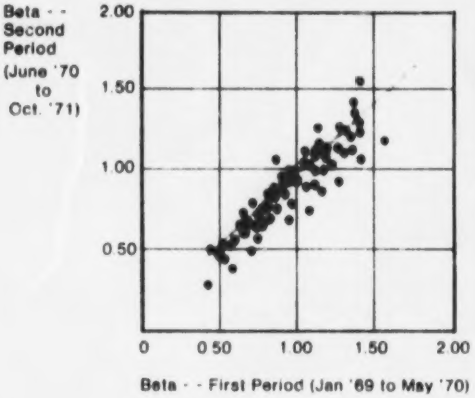
TABLE 5. CORRELATION OF 52-WEEK BETA FORECASTS WITH MEASURED VALUES FOR PORTFOLIOS OF N SECURITIES

1962-1970

Forecast for 52 Weeks Ended	Product Moment Correlations: N=				
	1	5	10	25	50
12/28/62	385	.711	.803	.933	.988
12/27/63	.492	.806	.866	.931	.963
12/25/64	.430	.715	.825	.945	.970
12/24/65	.451	.730	.809	.936	.977
12/23/66	.548	.803	.869	.952	.974
12/22/67	.474	.759	.830	.900	.940
12/20/68	.455	.732	.857	.945	.977
12/19/69	.556	.844	.922	.965	.973
12/18/70	.551	804	.888	.943	.985
Quadratic Mean	.486	.769	.853	939	.972

Source: Robert A. Levy [13], Table 2, p. 57.

EXHIBIT 10
INTERPERIOD BETA COMPARISON: DAILY DATA FOR 90 MUTUAL FUNDS



Source: Pogue and Conway [21]

Levy's tests indicated that an average of 24 per cent of the variation in second-period betas is explained by historical values. The betas of his portfolios, on the other hand, were much more predictable, the degree of predictability increasing with portfolio diversification. The results of the Pogue and Conway study and others show that fund betas, not unexpectedly, are not as stable as those for unmanaged portfolios. Nonetheless, two-thirds to three-quarters of the variation in fund betas can be explained by historical values.

The reader should remember that a significant portion of the measured changes in estimated beta values may not be due to changes in the true values, but rather to measurement errors. This observation is particularly applicable to individual security betas where the standard errors tend to be large.

8. Tests of The Capital Asset Pricing Model *

The major difficulty in testing the CAPM is that the model is stated in terms of investors' expectations and not in terms of realized returns. The fact that expectations are not always realized introduces an error term, which from a statistical point of view should be zero *on the average*, but not necessarily zero for any single stock or single period of time. After the fact, we would expect to observe

$$R_j = R_f + \beta_j (R_m - R_f) + \epsilon_j \,. \qquad (17a)$$

where R_j, $\dot{R}_m$, and R_f are the realized returns on stock j, the market index, and the riskless asset; and ϵ_j is the residual term.

If we observe the realized returns over a series of periods, the average security return would be given by

$$\bar{R}_j = \bar{R}_F + \beta_j (\bar{R}_M - \bar{R}_F) + \bar{\epsilon}_j \,. \qquad (17b)$$

where $\bar{R}_j$, $\bar{R}_M$, and $\bar{R}_F$ are the average realized returns on the stock, the market and the risk-free rate. If the CAPM is correct, the average residual term, $\bar{\epsilon}_j$, should approach zero as the number of periods used to compute the average becomes large. To test this hypothesis, we can regress the average returns, R_j, for a series of stocks ($j=1, \ldots, N$) on the stocks' estimated beta values, $\hat{\beta}_j$, during the period studied. The equation of the fitted line is given by

$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \mu_j \,. \qquad (18a)$$

where γ_0 and γ_1 are the intercept and slope of the line, and μ_j is the deviation of stock j from the line. By comparing Equations (17b) and (18a), we infer that if the CAPM hypothesis is valid, μ_j should equal $\bar{\epsilon}_j$ and hence should be small. Furthermore, it should be uncorrelated with $\hat{\beta}_j$, and hence we can also infer that γ_0 and γ_1 should equal $\bar{R}_F$ and $\bar{R}_M - \bar{R}_F$ respectively.

The hypothesis is illustrated in Exhibit 11. Each plotted point represents one stock's realized return versus the stock's beta. The vertical distances of the points from the CAPM theoretical line (also called the "market line") represent the mean residual returns, $\bar{\epsilon}_j$. Assuming the CAPM to be correct, the $\bar{\epsilon}_j$ should be uncorrelated with the $\hat{\beta}_j$ and thus the regression equation fitted to these points should be (1) linear, (2) upward sloping with slope equal to $\bar{R}_M - \bar{R}_F$, and (3) should pass through the vertical axis at the risk-free rate.

Expressed in risk-premium form, the equation of the fitted line is

$$\bar{r}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \mu_j \,. \qquad (18b)$$

where $\bar{r}_j$ is the average realized risk premium for stock j. Comparing Equation (18b) to the CAPM in risk-premium form [Equation (15)], the predicted values for γ_0 and γ_1 are 0 and $\bar{r}_m$, the mean market risk premium ($\bar{R}_M - \bar{R}_f$). Thus shifting to risk premiums changes the predicted value only for γ_0, but not for γ_1.

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*The material in this section was also prepared as an appendix to testimony to be delivered before the Federal Communications Commission by S.C. Myers and G.A. Pogue

Other Measures of Risk

The hypothesis just described is only true if beta is a complete measure of a stock's risk. Various alternative risk measures have been proposed, however. The most common alternative hypothesis states that expected return is related to the standard deviation of return—that is, to a stock's total risk, which includes both systematic and unsystematic components.

Which is more important in explaining average observed returns on securities, systematic or unsystematic risk? The way to find out is to fit an expanded equation to the data:

$$R_j = \gamma_0 + \gamma_1\hat{\beta}_j + \gamma_2(\hat{SE}_j) + \mu_j . \qquad (19)$$

Here $\hat{\beta}_j$ is a measure of systematic risk and $\hat{SE}_j$ a measure of unsystematic risk.[7] Of course, if the CAPM is exactly true, then γ_2 will be zero—that is, $\hat{SE}_j$ will contribute nothing to the explanation of observed security returns.

Tests of the Capital Asset Pricing Model

If the CAPM is right, empirical tests would show the following:

1. On the average, and over long periods of time, the securities with high systematic risk should have high rates of return.
2. On the average, there should be a linear relationship between systematic risk and return.
3. The slope of the relationship (γ_1) should be equal to the mean market risk premium ($\bar{R}_M - \bar{R}_F$) during the period used.
4. The constant term (γ_0) should be equal to the mean risk-free rate (R_F).
5. Unsystematic risk, as measured by $\hat{SE}_j$, should play no significant role in explaining differences in security returns.

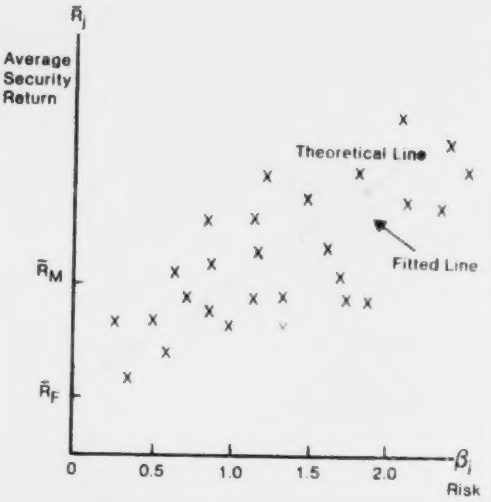
These predictions have been tested in several recent statistical studies. We will review some of the more important ones. Readers wishing to skip the details may proceed to the summary at the end of this section. We will begin by summarizing results from studies based on individual securities, and then we will turn to portfolio results.

Results of Tests Based on Securities

The Jacob Study. The Jacob study [9] deals with the 593 New York Stock Exchange stocks for which there is complete data from 1946 to 1965. Regression analyses were performed for the 1946-55 and 1956-65 periods, using both monthly and annual security returns. The relationship of mean security returns and beta values is shown in Table 6. The last two columns of the table give the theoretical values for the coefficients, as predicted by the CAPM.

Exhibit 11

RELATIONSHIP BETWEEN AVERAGE RETURN ($\bar{R}_j$) AND SECURITY RISK (β_j)



The results show a significant positive relationship between realized return and risk during each of the 10-year periods. For example, in 1956-65 there was a 6.7 per cent per year increase in average return for a one-unit increase in beta. Although the relationships shown in Table 6 are all positive, they are weaker than those predicted by the CAPM. In each period γ_1 is less and γ_0 is greater than the theoretical values.

The Miller-Scholes Study. The Miller-Scholes research [19] deals with annual returns for 631 stocks during the 1954-63 period. The results of three of their tests are reported in Table 7. The tests are (1) mean return versus beta, (2) mean return versus unsystematic ri[illegible] $\hat{SE}_j)^2$, and (3) mean return versus both beta and unsystematic risk.

The results for the first test show a significant positive relationship between mean return and beta. A one-unit increase in beta is associated with a 7.1 per cent increase in mean return.

The results for the second test do not agree with the CAPM's predictions. That is, high unsystematic risk is apparently associated with higher realized returns. However, Miller and Scholes show that this correlation may be largely spurious (i.e., it may be due to statistical sampling problems). For example, a substantial positive correlation exists between beta and $(S\hat{E}_j)^2$. Thus unsystematic risk will appear to be significant in tests from which beta has been omitted, even though it may be unimportant to the pricing of securities. This sort of statistical correlation need not imply a causal link between the variables.

Test number (3) includes both beta and $(S\hat{E}_j)^2$ in the regression equation. Both are found to be significantly positively related to mean return. The

TABLE 6. RESULTS OF JACOB'S STUDY

$$\bar{r}_j = \gamma_0 + \gamma_1\hat{\beta}_j + \mu_j$$

Tests Based on 593 Securities

Period	Return Interval	Regression Results[a]			Theoretical Values	
		$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2	$\gamma_0=0$	$\gamma_1 = R_M - R_F$
46-55	Monthly	0.80	0.30 (0.07)[b]	0.02	0	1.10
	Yearly	8.9	5.10 (0.53)	0.14	0	14.4
56-65	Monthly	0.70	0.30 (0.06)	0.03	0	0.8
	Yearly	6.7	6.7 (0.53)	0.21	0	10.8

(a) Coefficient units are: monthly data, percent per month; annual data, percent per year.

(b) Standard error.

Source: Jacob [9], Table 3, pp. 827-828.

TABLE 7. RESULTS OF THE MILLER AND SCHOLES STUDY

$$R_j = \gamma_0 + \gamma_1\hat{\beta}_j + \gamma_2(S\hat{E}_j)^2 + \mu_j$$

Annual Rates of Return 1954-1963
Tests Based on 631 Securities

Regression Results[a]				Theoretical Values		
$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	R^2	γ_0	γ_1	γ_2
12.2 (0.7)[b]	7.1 (0.6)		0.19	2.8	8.5	0
16.3 (0.4)		39.3 (2.5)	0.28	2.8	8.5	0
12.7 (0.6)	4.2 (0.6)	31.0 (2.6)	0.33	2.8	8.5	0

(a) Units of Coefficients: per cent per year.
(b) Standard error.

Source: Miller and Scholes [19], Table 1B, p. 53.

inclusion of $(\hat{SE}_j)^2$ has somewhat weakened the relationship of return and beta, however. A one-unit increase in beta is now associated with only a 4.2 per cent increase in mean return.

The interpretation of these results is again complicated by the strong positive correlation between beta and $(\hat{SE}_j)^2$, and by other sampling problems.[8] A significant portion of the correlation between mean return and $(\hat{SE}_j)^2$ may well be a spurious result. In any case, the results do show that stocks with high systematic risk tend to have higher rates of return.

Results for Tests Based on Portfolio Returns

Tests based directly on securities clearly show the significant positive correlation between return and systematic risk. Such tests, however, are not the most efficient method of obtaining estimates of the magnitude of the risk-return tradeoff. The tests are inefficient for two reasons.

The first problem is well known to economists. It is called "errors in variables bias" and results from the fact that beta, the independent variable in the test, is typically measured with some error. These errors are random in their effect—that is, some stocks' betas are overestimated and some are underestimated. Nevertheless, when these estimated beta values are used in the test, the measurement errors tend to attenuate the relationship between mean return and risk.

By carefully grouping the securities into portfolios, much of this measurement error problem can be eliminated. The errors in individual stocks' betas cancel out so that the portfolio beta can be measured with much greater precision. This in turn means that tests based on portfolio returns will be more efficient than tests based on security returns.

The second problem relates to the obscuring effect of residual variation. Realized security returns have a large random component, which typically accounts for about 70 per cent of the variation of return. (This is the diversifiable or unsystematic risk of the stock.) By grouping securities into portfolios, we can eliminate much of this "noise" and thereby get a much clearer view of the relationship between return and systematic risk.

It should be noted that grouping does not distort the underlying risk-return relationship. The relationship that exists for individual securities is exactly the same for portfolios of securities.

Friend and Blume Studies. Professors Friend and Blume [3,8] have conducted two interrelated risk-return studies. The first examines the relationship between long-run rates of return and various risk measures. The second is a direct test of the CAPM

In the first study [8], Friend and Blume constructed portfolios of NYSE common stocks at the beginning of three different holding periods. The periods began at the ends of 1929, 1948, and 1956. All stocks for which monthly rate-of-return data

TABLE 8. RESULTS OF FRIEND-BLUME STUDY

Returns from a yearly revision policy for stocks classified by beta for various periods

	Holding Period					
	1929-1969		1948-1969		1956-1969	
Portfolio No.	Beta	Mean Return %	Beta	Mean Return %	Beta	Mean Return %
1	0.19	0.79	0.45	0.99	0.28	0.95
2	0.49	1.00	0.64	1.01	0.51	0.98
3	0.67	1.10	0.76	1.25	0.66	1.12
4	0.81	1.28	0.85	1.30	0.80	1.18
5	0.92	1.26	0.94	1.35	0.91	1.17
6	1.02	1.34	1.03	1.37	1.03	1.14
7	1.15	1.42	1.12	1.32	1.16	1.10
8	1.29	1.53	1.23	1.33	1.30	1.18
9	1.49	1.55	1.36	1.39	1.48	1.15
10	2.02	1.59	1.67	1.36	1.92	1.10

Monthly arithmetic mean returns
Source: Friend and Blume [8], Table 4, p. 10.

could be obtained for at least four years preceding the test period were divided into 10 portfolios. The securities were assigned on the basis of their betas during the preceding four years—the 10 per cent of securities with the lowest betas to the first portfolio, the group with the next lowest betas to the second portfolio, and so on.

After the start of the test periods, the securities were reassigned annually. That is, each stock's estimated beta was recomputed at the end of each successive year, the stocks were ranked again on the basis of their betas, and new portfolios were formed. This procedure kept the portfolio betas reasonably stable over time.

The performance of these portfolios is summarized in Table 8. The table gives the arithmetic mean monthly returns and average beta values for each of the 10 portfolios and for each test period.

For the 1929-69 period, the results indicate a strong positive association between return and beta. For the 1948-69 period, while higher beta portfolios had higher returns than portfolios with lower betas, there was little difference in return among portfolios with betas greater than 1.0. The 1956-69 period results do not show a clear relationship between beta and return. On the basis of these and other tests, the authors conclude that NYSE stocks with above average risk have higher returns than those with below average risk, but that there is little payoff for assuming additional risk within the group of stocks with above average betas.

In their second study [3], Blume and Friend used monthly portfolio returns during the 1955-68 period to test the CAPM. Their tests involved fitting the coefficients of Equation (18a) for three sequential periods: 1955-59, 1960-64, and 1965-68. The authors also added a factor to the regression equation to test for the linearity of the risk-return relationship.[9]

The values obtained for γ_0 and γ_1 are not in line with the Capital Asset Pricing Model's predictions, however. In the first two periods, γ_0 is substantially larger than the theoretical value. In the third period, the reverse situation exists, with γ_0 substantially less than predicted. These results imply that γ_1, the slope of the fitted line, is less than predicted in the first two periods and greater in the third.[10] Friend and Blume conclude that "the comparisons as a whole suggest that a linear model is a tenable approximation of the empirical relationship between return and risk for NYSE stocks over the three periods covered."[11]

Black, Jensen, and Scholes. This study [1] is a careful attempt to reduce measurement errors that would bias the regression results. For each year from 1931 to 1965, the authors grouped all NYSE stocks into 10 portfolios. The number of securities in each portfolio increased over the 35-year period from a low of 58 securities per portfolio in 1931 to a high of 110 in 1965.

Month-by-month returns for the portfolios were computed from January 1931 to December 1965. Average portfolio returns and portfolio betas were computed for the 35-year period and for a variety of sub-periods. The results for the complete period are shown in Table 9. The average monthly portfolio returns and beta values for the 10 portfolios are plotted in Exhibit 12. The results indicate that over the complete 35-year period, average return increased by approximately 1.08 per cent per month (13 per cent per year) for a one-unit in-

TABLE 9. RESULTS OF BLACK-JENSEN-SCHOLES STUDY

$$R_p = \gamma_0 + \gamma_1\hat{\beta}_p + \mu_p$$

1931-1965

Tests Based on 10 Portfolios
(Averaging 75 Stocks per Portfolio)

Regression Results[a]			Theoretical Values	
$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2	$\gamma_0 = R_F$	$\gamma_1 = R_M - R_F$
0.519 (0.05)[b]	1.08 (0.05)	0.90	0.16	1.42

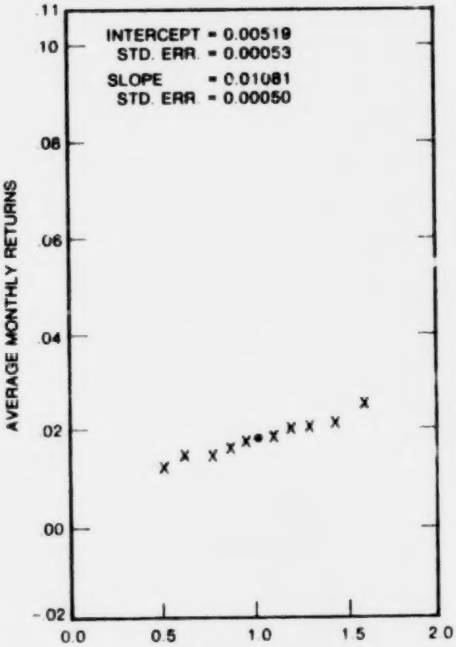
(a) Units of Coefficients: per cent per month.

(b) Standard error.

Source: Black, Jensen, and Scholes [1], Table 4, p. 98, and Figure 7, p. 104.

crease in beta. This is about three-quarters of the amount predicted by the CAPM. As Exhibit 12 shows, there appears to be little reason to question the linearity of the relationship over the 35-year period.

Exhibit 12
RESULTS OF BLACK, JENSEN AND SCHOLES STUDY
1931 - 1965



Average monthly returns versus systematic risk for the 35-year period 1931-1965 for ten portfolios and the market portfolio

Source: Black, Jensen, and Scholes [1], Figure 7, p. 104

Black, Jensen, and Scholes also estimated the risk-return tradeoff for a number of subperiods.[12] The slopes of the regression lines tend in most periods to understate the theoretical values, but are generally of the correct sign. Also, the subperiod relationships appear to be linear.

This paper provides substantial support for the hypothesis that realized returns are a linear function of systematic risk values. Moreover, it shows that the relationship is significantly positive over long periods of time.

Fama and MacBeth. Fama and MacBeth [6] have extended the Black-Jensen-Scholes tests to include two additional factors. The first is an average of the β_j^2 for all individual securities in portfolio p, designated $\hat{\beta}_p^2$. The second is a similar average of the residual standard deviations ($S\hat{E}_j$) for all stocks in portfolio p, designated $S\hat{E}_p$. The first term tests for nonlinearities in the risk-return relationship, the second for the impact of residual variation.

The equation of the fitted line for the Fama-MacBeth study is given by

$$\bar{R}_p = \gamma_0 + \gamma_1\hat{\beta}_p + \gamma_2\hat{\beta}_p^2 + \gamma_3 S\hat{E}_p + \mu_p \, . \qquad (20)$$

where, according to the CAPM, we should expect γ_2 and γ_3 to have zero values.

The results of the Fama-MacBeth tests show that while estimated values of γ_2 and γ_3 are not equal to zero for each interval examined, their average values tend to be insignificantly different from zero. Fama and MacBeth also confirm the Black-Jensen-Scholes result that the realized values of γ_0 are not equal to $\bar{R}_f$, as predicted by the CAPM.

Summary of Test Results

We will briefly summarize the major results of the empirical tests.

1. The evidence shows a significant positive relationship between realized returns and systematic risk. However, the slope of the relationship (γ_1) is usually less than predicted by the CAPM.
2. The relationship between risk and return appears to be linear. The studies give no evidence of significant curvature in the risk-return relationship.
3. Tests that attempt to discriminate between the effects of systematic and unsystematic risk do not yield definitive results. Both kinds of risk appear to be positively related to security returns. However, there is substantial support for the proposition that the relationship between return and unsystematic risk is at least partly spurious—that is, it partly reflects statistical problems rather than the true nature of capital markets.

Obviously, we cannot claim that the CAPM is absolutely right. On the other hand, the empirical tests do support the view that beta is a useful risk measure and that high beta stocks tend to be priced so as to yield correspondingly high rates of return.

9. Measurement of Investment Performance

The basic concept underlying investment performance measurement follows directly from the risk-return theory. The return on managed portfolios, such as mutual funds, can be judged relative to the returns on unmanaged portfolios at the same degree of investment risk. If the return exceeds the standard, the portfolio manager has performed in a superior way, and vice versa. Given this, it remains to select a set of "benchmark" portfolios against which the performance of managed portfolios can be evaluated.

Performance Measures Developed from the Capital Asset Pricing Model

The CAPM provides a convenient and familiar standard for performance measurement; the benchmark portfolios are simply combinations of the riskless asset and the market index. The return standard for a mutual fund, for example, with beta equal to β_P, is equal to the risk-free rate ($\bar{R}_F$) plus β_P times the average realized risk premium on the market ($\bar{R}_M - \bar{R}_F$). Thus the return on the performance standard ($\bar{R}_S$) is given by

$$\bar{R}_S = \bar{R}_F + \beta_P(\bar{R}_M - \bar{R}_F) , \qquad (21)$$

where $\bar{R}_M$ and $\bar{R}_F$ are the arithmetic average returns on the market index and riskless asset during the evaluation period. The performance measure, designated α_p, is equal to the difference in average returns between the fund and its standard; that is,

$$\alpha_p = \bar{R}_P - \bar{R}_S , \qquad (22)$$

where $\bar{R}_P$ is the arithmetic average return on the fund. Under the CAPM assumption, the expected values of $\bar{R}_P$ and $\bar{R}_S$ are the same; therefore the expected value for the performance measure $\hat{\alpha}_p$ is zero. Managed portfolios with positive estimated values for α_p have thus outperformed the standard, and vice versa. Estimated values of alpha ($\hat{\alpha}_p$) are determined by regressing the portfolio risk premiums on the corresponding market risk premiums.

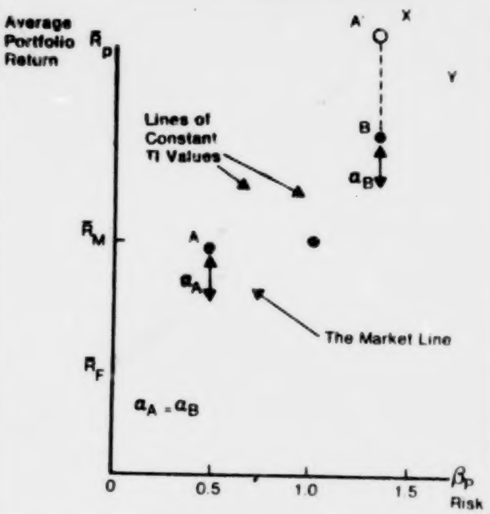
The interpretation of the estimated alpha must take into consideration possible statistical measurement errors. As we discussed in Section 7, the standard error of alpha (SE_α) is an indication of the extent of the possible measurement error. The larger the standard error, the less certain we can be that measured alpha is a close approximation of the true value.[13]

A measure of the degree of statistical significance of the estimated alpha value is given by the ratio of the estimated alpha to its standard error. The ratio, designated as t_α, is given by

$$t_\alpha = \hat{\alpha}_p / SE_\alpha . \qquad (23)$$

Exhibit 13

RELATIONSHIP BETWEEN THE JENSEN AND TREYNOR MEASURES OF INVESTMENT PERFORMANCE



Symbols: $\bar{R}_M$ = Return on Market Index
$\bar{R}_F$ = Risk-free Rate of Interest
A,B = Managed Portfolios
A' = Portfolio A Levered to Same Beta as Portfolio B

The statistic t_α gives a measure of the extent to which the true value of alpha can be considered to be different from zero. If the absolute value of t_α is large, then we have more confidence that the true value of alpha is different from zero. Absolute values of t_α in excess of 2.0 indicate a probability of less than about 2.5 per cent that the true value of alpha is zero.

These methods of performance measurement were originally devised by Michael Jensen [10,11] and have been widely used in many studies of investment performance, including that of the recent SEC Institutional Investor Study [21].

A performance measure closely related to the Jensen alpha measure was developed by Jack L. Treynor [24]. The Treynor performance measure (designated TI)[14] is given by

$$TI = \alpha/\beta . \qquad (24)$$

The difference between the α and TI performance measures is simply that the fund alpha value has been divided by its estimated beta. The effect, however, is significant, eliminating a so-called "leverage bias" from the Jensen alpha measures. This is illustrated in Exhibit 13.

Funds A and B in Exhibit 13 have the same alpha values. (The alphas are equal to the vertical distance on the diagram between the funds and the market line.) By combining portfolio B with the riskless rate (that is, by borrowing or lending at R_f), any return-risk combination along line Y can be obtained. But such points are clearly dominated by combinations along line X—attainable by borrowing or lending combined with fund A. As Exhibit 13 shows, the alpha for fund A, when levered to the same beta as fund B (Point A'), dominates the latter's alpha value.

The Treynor measure eliminates this leverage effect. All funds which lie along a line (such as X or Y) have the same TI value; therefore borrowing or lending combined with any fund outcome will not increase (or decrease) its performance measure. The Treynor measure thus permits direct performance comparisons among funds with differing beta values.

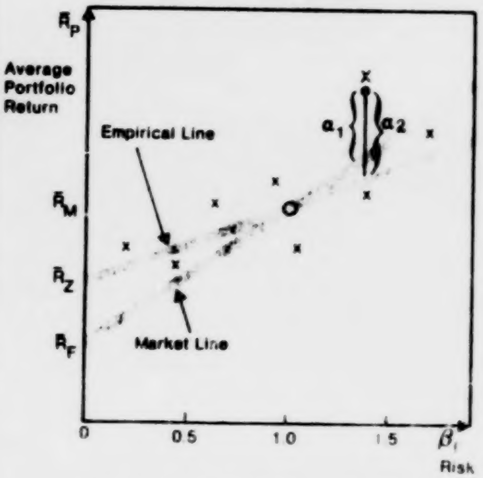
Problems with the Market Line Standard

The tests of the CAPM summarized in Section 8 indicate that the average returns over time on securities and portfolios tend to deviate from the predictions of the model. Though the observed risk-return relationships seem to be linear, they are generally flatter than predicted by the CAPM, implying that the tradeoff of risk for return is less than predicted.

This evidence raises some question as to whether the CAPM market line provides the best benchmark for performance measurement and suggests instead that other benchmark portfolios may be more appropriate. For example, under certain conditions, the "empirical" risk-return lines developed by Black, Jensen, and Scholes [1] and others would seem to be a reasonable alternative to the CAPM market line standard. This might be the case if the portfolio for which performance is being measured were restricted to exactly the same set of investment options used to create the empirical standard, that is, if the portfolio were fully invested in common stock and could not use leverage to increase its beta value. For such a portfolio it would seem appropriate to measure performance relative to the empirical line, as opposed to the market line.

Exhibit 14

MEASUREMENT OF INVESTMENT PERFORMANCE: MARKET LINE VERSUS EMPIRICAL STANDARD



Symbols:
$\bar{R}_M$ = Return on Market Index
$\bar{R}_Z$ = Return on Zero Beta Portfolio
$\bar{R}_F$ = Risk-Free Rate of Interest
X = Investment Portfolios
O = Market Index

A comparison of these standards is illustrated in Exhibit 14. The market line performance measure (designated as α_1 in Exhibit 14) is equal to the vertical distance from the portfolio to the market line. The empirical line measure (designated α_2) is the vertical distance from the portfolio to the empirical line. Since ideally all the stocks used to develop the empirical line are contained in the market index, the empirical line, like the market line, would be expected to have a return equal to market return, $\bar{R}_M$, for beta equal to 1.0. The intercepts on the return axis, however, are typically different for the two lines. The market line intercept, by definition, is equal to the average risk-free rate. The empirical line intersects the return axis at a point different from $\bar{R}_f$, and typically above it. This intercept equals the average return on a portfolio with "zero beta", designated $\bar{R}_Z$. The existence of a long-run average return on the zero beta portfolio that differs from the riskless rate is a clear violation of the predictions of the CAPM. As of this time, there is no clear theoretical understanding of the reason for this difference.

To summarize, empirically based performance standards could, under certain conditions, provide alternatives to those of the CAPM market line standard. However, the design of appropriate empirical standards requires further research. In the interim, the familiar market line benchmarks can provide useful information regarding performance, although the information should not be regarded as being very precise.[15] ∎

Footnotes

1. From this point on, systematic risk will be referred to simply as "risk." Total risk will be referred to as "total risk."
2. We use the term portfolio in a general sense, including the case where the investor holds only one security. Since portfolio return and (systematic) risk are simply weighted averages of security values, risk-return relationships which hold for securities must also be true for portfolios, and vice versa.
3. The sample was picked to give the broadest possible range of security beta values. This was accomplished by ranking all NYSE securities with complete data from 1945-70 by their estimated beta values during this period. We then selected every 25th stock from the ordered list. The data was obtained from the University of Chicago CRSP (Center for Research in Security Prices) tape.
4. The commercial paper results in Table 3 are rates of return, not risk premiums. The risk premiums would equal zero by definition.
5. Correlation studies of this type tend to produce a conservative picture of the degree of beta coefficient stationarity. This results from the fact that it is not possible to correlate the true beta values but only estimates which contain varying degrees of measurement error. Measurement error would reduce the correlation coefficient even though the underlying beta values were unchanged from period to period.
6. These results are consistent with those found by N. Mains in a later study [16]. Mains correlated adjacent calendar-year betas for a sample of 99 funds for the period 1960 through 1971. The betas were based on weekly returns. The average correlation coefficient for 11 tests was 0.788, with individual values ranging from a low of 0.614 to a high of 0.871.
7. SE$_j$ is an estimate of the standard error of the residual term in Equation (17a). Thus it is the estimated value for $\sigma(e_j)$, the unsystematic risk term defined in Equation (8). See column (6) of Tables 3 and 4 for typical values for securities and mutual funds.
8. For example, skewness in the distributions of stock returns can lead to spurious correlations between mean return and SE$_j$. See Miller and Scholes [19], pp. 66-71.
9. Their expanded test equation is
$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_2 (\hat{\beta}_j)^2,$$
where, according to the CAPM, the expected value of γ_2 is zero.
10. Table 1, p. 25, of Blome and Friend [3] presents period-by-period regression results.
11. Blume and Friend [3], p. 26.
12. Figure 6 of Black, Jensen, and Scholes [1], pp. 101-103, shows average monthly returns versus systematic risk for 17 nonoverlapping 2-year periods from 1932 to 1965.
13. See columns 2 and 3 of Table 4 for typical mutual fund $\hat{\alpha}$ and SE$_\alpha$ values.
14. Treynor's work preceded that of Jensen. In a discussion of Jensen's performance measure [26], Treynor showed that his measure (as originally presented in [25]) was equivalent to
$$\text{TI} = R_f - \alpha/\beta.$$
Since R_f is a constant, the TI index for ranking purposes is equivalent to that given in Equation (24).
15. There are a number of excellent references for further study of portfolio theory. Among these we would recommend books by Richard A. Brealey [4], Jack Clark Frances [7], and William F. Sharpe [24]. For a more technical survey of the theoretical and empirical literature, see Jensen [12].

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IV. THE RELATIONSHIP BETWEEN REAL AND FINANCIAL MEASURES OF RISK

Section IV is concerned with the real determinants of the risk of common stocks as measured by their beta values. By real determinants are meant variables which can be measured directly from a firm's balance sheets and income statements. The beta risk measure is the key variable of the Capital Asset Pricing Model (CAPM) developed by Sharpe [26], Lintner [9], and Mossin [17]. Their model is given by

$$E(R_j) = R_F + \beta_j \left(E(R_M) - R_F\right) \qquad (3\text{-}5)$$

where

$E(R_j)$ = the expected one-period rate of return on stock j

$E(R_M)$ = the expected one-period return on the market portfolio

R_F = the one-period risk-free rate of interest

β_j = the beta value for stock j

The model states that the expected rate of return on the firm's common stock (a financial asset) is a linear function of its market risk measure, beta. The model deals only with the returns on financial assets. It says nothing about how the expected returns or beta values are related to characteristics of firms' real assets that can be measured directly from accounting (i.e., book) data. In other words, the CAPM says nothing about the real determinants of beta.

As will be shown below, there have been several empirical investigations of the real or book characteristics of high-risk firms. The results are mostly consistent with intuition, in that (various proxies for) earnings variability, growth, cyclicality, and financial leverage are significantly correlated with beta. However, there is not much theory having more than intuitive content. The missing link is a dynamic model of a firm's earnings behavior, growth, and market valuation. The development of a fully rigorous and operational theory of the real determinants of asset values is a difficult task which, to date, has not been accomplished.

Section IV is divided into three segments. The first, segment (a) presents a very complete survey of the empirical literature through mid-1973. It was prepared by Professor Stewart C. Myers of MIT as Part I of a paper entitled, "The Relationship between Real and Financial Measures of Risk and Return" [22]. In segment (b) this survey is brought up to date (mid-1975). In segment (c) the current state of this research is summarized. Readers interested only in the conclusions and not the details can skip directly to segment (c).

(a) Empirical Studies on the Real Determinants of Beta (Through Mid-1973)* (Excerpt from a Paper [22] by Professor Stewart C. Myers)

This part reviews and summarizes all the empirical work known to me relating to the real determinants of stocks' systematic risk. However, it is obviously impossible to provide a full description of each study's experimental design. The following discussion is limited to the main ideas and results.

Beaver, Kettler, and Scholes

Beaver, Kettler and Scholes (BKS [2]) were the empirical pioneers, although Ball and Brown's work [1] preceded theirs. However, the Ball and Brown paper is mostly devoted to other, broader issues, and the part investigating the real determinants of systematic risk is much less ambitious and informative than the BKS study. In any event, Ball and Brown's results were confirmed by BKS.

BKS used seven accounting variables that intuition or tradition suggest are associated with high-risk firms.

1. Dividend payout, defined as total dividends per share paid by the firm over a nine-year period, divided by total earnings per share over the same

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* Footnotes and references for this section appear at the end of the article. Reproduced with permission of Author and Publisher.

period. Two arguments can be made for including payout as an independent variable. First, most firms attempt to achieve stable (or stable growth in) dividends, and in normal times regard them almost as fixed liability. Nevertheless, earnings have to cover dividends, at least in the long run. The higher the variance of earnings, the lower must the normal dividend be set in order to keep the probability of "trouble" (earnings less than the normal dividend) acceptably low. Thus dividend payout should be a proxy for management's uncertainty about future earnings. Second, firms which grow rapidly usually retain a greater proportion of earnings. Since there is a long tradition in the literature associating high growth with high risk, low dividend payout should be a proxy for high risk.[1]

2. *Growth*, measured by the log of the five-year change in net book assets. Again, the rationale is the traditional association between rapid growth and high business risk.
3. *Financial leverage*, measured by the ratio of book debt to net book assets. Given business risk, the CAPM predicts a positive linear relationship between beta and the ratio of the *market* value of debt to the total market value of the firm (debt plus equity).[2]

However, this relationship will be difficult to observe if firms with low business risk find they can issue more debt.[3] In the limiting case where firms keep the sum of business and financial risk constant, there will be no evident relationship between the debt ratio and beta (until other variables successfully account for differences in business risk).

4. Liquidity, measured by ratio of current assets to current liabilities. This is widely used as measure of solvency by creditors.

5. Size, measured by the log of net book assets. Casual observation suggests a relationship between size and safety, and to the extent that size reflects diversification of activity, theory predicts that large firms will have lower total risk. There is no rigorous theory predicting that large firms have lower systematic risk, however.

6. Earnings variability, measured by the standard deviation of earnings per share over nine years, where each year's net earnings available to common is normalized by dividing by the value of the firm's stock at the end of the preceding year. It makes sense to predict that firms with highly volatile earnings will have highly volatile stock prices.

This means high total risk, however, not necessarily high systematic risk. There is an empirical correlation between total and systematic risk, but this is to be expected, simply because the former includes the latter. It is not true that increased total risk necessarily means increased systematic risk. Also, note that this variable is partly dependent on variable 3, financial leverage: the more debt the firm uses, the greater the variance of earnings available to common stockholders.

7. The "Earnings Beta", which is the slope coefficient of the regression of the firm's net earnings (again normalized by the preceding period's stock value) on the average normalized earnings for the entire sample of firms. Thus it is a measure of cyclicality, that is, the extent to which fluctuations in the firm's earnings are correlated with fluctuations in earnings of firms generally. Since stock prices clearly respond to earnings, both individually and generally, the earnings beta and the stock beta ought to be strongly related.

I suspect that most readers will find this list of variables intuitively reasonable. The intuitive concept of risk, however, includes many things that are not necessarily related to risk as defined by the CAPM. A rigorous *a priori* hypothesis can be made with respect to only one of the seven variables, namely financial leverage. A less rigorous but very plausible case can be made for variable 7, the "Accounting Beta". As for the other five variables, there is no theory.

BKS tested the relationships between these variables and beta by cross-sectional tests on a sample of 307 firms for which complete accounting and stock price data were available for the period 1947-65. Actually, four separate tests were run. First, the data were split into two subperiods, 1947-56 and 1957-65. Within each period, tests were performed both on individual securities and on five-security portfolios. The portfolios were formed by ranking the stocks on basis of the relevant accounting variable, and then assigning firms ranked 1 - 5 to portfolio 1, those ranked 6 - 10 to portfolio 2, etc. Tests were then performed on portfolio averages of the accounting variables and betas.[4]

For each of the four cases, pair-wise rank correlations were calculated between each of the seven variables and contemporaneous firm or portfolio betas. The results are shown in Table 1. It is gratifying that leverage and the accounting beta show the strong positive correlations expected on theoretical grounds. (In this case a rank correlation of ±0.14 is significant at approximately the one percent confidence level.) Earnings variability and payout also have

Table 1

CONTEMPORANEOUS ASSOCIATION BETWEEN MARKET-DETERMINED MEASURE OF RISK AND SEVEN ACCOUNTING RISK MEASURES[a]

Variable	Period One (1947-56)		Period Two (1957-65)	
	Individual Level	Portfolio[b] Level	Individual Level	Portfolio[b] Level
Payout	-.49 (-.50)	-.79 (-.77)	-.29 (-.24)	-.50 (-.45)
Growth	.27 (.23)	.56 (.51)	.01 (.03)	.02 (.07)
Leverage	.23 (.23)	.41 (.45)	.22 (.25)	.48 (.56)
Liquidity	-.13 (-.13)	-.35 (-.44)	.05 (-.01)	.04 (-.01)
Size	-.06 (-.07)	-.09 (-.13)	-.16 (-.16)	-.30 (-.30)
Earnings Variability	.66 (.58)	.90 (.77)	.45 (.36)	.82 (.62)
Accounting Beta	.44 (.39)	.68 (.67)	.23 (.23)	.46 (.46)

[a] Rank correlation coefficients appear in top row, and product-moment correlations appear in parentheses in bottom row.

[b] The portfolio correlations are based on 61 portfolios of 5 securities each.

Source: Beaver, Kettler and Scholes [2], p. 669.

significant correlations in the expected direction. Size has a somewhat weaker correlation. Finally, growth and liquidity, although they were significantly correlated with beta in the first subperiod, are totally unrelated in the second. In view of this, it is hard to put much faith in these two variables.

Viewed as a whole, the results are encouraging. Theory, what there is of it, is supported, and intuition is confirmed. However, one can obtain significant associations between variables without actually explaining very much, and to some extent that is the case here. BKS ran a second test in which they attempted to predict the 1957-65 betas by using the accounting variables from the earlier 1947-56 period. The best explanator of the 1947-56 betas, as determined by regression analysis, is a linear combination of dividend payout, asset growth, and earnings variability. This procedure explains 44.7 percent of the variance in the first period's betas.[5] However, when this equation is used to predict the 1957-65 betas, using account variables from the first period, only 24 percent of the variance is explained. This is not too impressive, considering that 21 percent is explained by the "naive" prediction that the 1957-65 beta would be equal to the 1947-56 beta. Nevertheless, it is some improvement. Moreover, it must be remembered that 100 percent prediction accuracy is impossible. Even if we knew the exact, true betas of all firms for the second period, we could not predict the measured betas, for that would require an exact *a priori* prediction of the sampling errors encountered in estimation.

White

White [15] followed broadly the same methodology as BKS, but with important differences in variable definition and in the way the tests were carried out. He hypothesized three main factors characteristic of high-beta firms:

1. High debt ratio. This variable's effect on beta is predicted by theory and supported by the BKS results. However, White defined the debt ratio in terms of market values,[6] which is the specification called for by theory. BKS considered only variables that could be derived from accounting data, and so restricted their tests to the book debt ratio.
2. Rapid growth in sales or operating earnings, measured by the log of the relative change of the variable over the period examined. BKS examined growth in book assets but obtained mixed results.
3. High asset beta, defined as the slope coefficient of a regression of each firm's percentage change in sales on the contemporaneous percentage change in gross national product. This is analogous to BKS's "accounting beta" in that each measures firms'

cyclicality. However, there are two important differences. First, the index used is a national one, not simply an average taken over the firms in the sample under consideration. Since the investor is concerned with the covariance of each stock's return with the rate of return on the entire market (and in principle with its covariance with changes in aggregate national wealth), he should likewise be concerned with the covariance of each firm's income with national income. It is true that the average earnings of any reasonably large sample of firms will be highly correlated with national income, but errors will necessarily enter if the sample average is used.

The second difference between White and BKS's definition of cyclicality is that White used sales rather than earnings. It is true that investors are more concerned with earnings than sales per se. But it is expected earnings which determine stock prices, and changes in sales may be a better proxy for changes in expected earnings than is the one-period change in accounting earnings. Accounting procedures induce various lags and biases in reported income, for example.

Aside from differences in variable definition, White's study differs from BKS's in the use of relatively large portfolios of securities as the basis for his cross-sectional tests. This has become standard operating procedure in tests of the CAPM, since it vastly reduces errors in measurements of individual firms' betas; these errors otherwise attenuate the observed relationship between firms' betas and their average returns. The aim in the present context is to reduce errors both in measuring beta and in measuring the three hypothesized determinants of beta.

The regressions summarized in Table 2 are typical of White's results. In these tests the 210 securities in White's sample were grouped into 10 portfolios of 31 securities each. (The 31 stocks with the highest betas were put in portfolio 1, the 31 with the next highest betas in portfolio 2, etc.) Then four cross-sectional regressions were run, using as independent variables various combinations of the portfolios' asset betas, debt ratios, and rates of sales or earnings growth. As is obvious from the table, White found significant relationships, in the expected direction, for all variables. He also explained a much higher proportion of the variance in the betas, although it must be remembered that in this test there are only 10 portfolio betas to be explained.

These tests were successfully repeated for various subperiods within the 1951-68 period,[7] but I will not attempt to present all the results here. It should be noted, however,

Table 2

WHITE'S REGRESSION RESULTS FOR 10 PORTFOLIOS OF 31 SECURITIES EACH, 1951-68

Regression	Variable				
	Constant	Asset Beta	Market Debt Ratio	Sales Growth	Earnings Growth
1	0.505	0.325			
(R^2 = .71)	(4.54)[a]	(4.38)			
2	0.153	0.247	2.45		
(R^2 = .85)	(0.969)	(3.88)	(2.62)		
3	-0.347	0.274	2.35	7.16	
(R^2 = .96)	(-2.31)	(7.65)	(4.56)	(4.10)	
4	-0.294	0.220	1.69		7.68
(R^2 = .97)	(-2.53)	(7.22)	(3.62)		(5.03)

[a] T-statistics in parentheses below coefficients.

Source: White [15], Table 4.6.

that the coefficients of the growth variables dropped to insignificant levels when White repeated his tests for the 1961-68 subperiod. This matches BKS's findings. However, since the asset beta—the "cyclicality" variable—and the debt ratio both have a strong positive relationship with beta, it is fair to say that White's results support the conclusions of the BKS study, and of course vice versa.

Gonedes

Gonedes's work [4] provides additional, and somewhat discouraging, evidence on the importance of BKS's accounting beta as a determinant of firms' stock betas. He dealt with a random sample of 99 firms for the period 1946-67, and three seven-year subperiods, 1946-52, 1953-59, and 1961-67. (The data for 1960 were reserved for various statistical tests which will not be reviewed here.) The accounting betas were measured by regressions of the form

$$\tilde{X}_j = a + b_j \tilde{X}_m + C_j \tilde{X}_I + \tilde{e}_j \tag{2}$$

where:

$\tilde{X}_j$ = firm j's earnings

$\tilde{X}_m$ = aggregate earnings of all firms for which complete data were available over the 1946-68 period

$\tilde{X}_I$ = that part of the aggregate earnings of all other firms in j's industry which could not be explained by X_M

$\tilde{e}_j$ = an error term reflecting unsystematic risk

The 99 firms in the sample were randomly selected from industries for which adequate indexes could be constructed.

Regressions were run using net income (after interest and taxes), net income normalized by net book assets, and first differences of these two variables. As it turned out only the first differences gave significant results.

Gonedes tested for correlation between stock betas and accounting betas measured in terms of scaled net income and found no significant relationship for any subperiods. This is exactly counter to the BKS results. However, some positive relationship was found for accounting betas derived from first differences in scaled income. This is again counter to BKS, who found that this procedure gave poorer results for their sample.[8]

Table 3 shows the correlations obtained using accounting betas derived from first differences of scaled net income. Significant relationships are found for the 1946-52 and 1953-59 subperiods when accounting betas are derived from the full 21 years of data, but the relationship disappears in the 1960's. There is no relationship at all when the accounting betas are measured from seven years' data, but this probably indicates only that accounting betas are hard to measure with a handful of observations.

Table 3

CORRELATION BETWEEN ACCOUNTING AND STOCK BETAS —GONEDES'S RESULTS

(Accounting betas derived from first differences in net income scaled by net book assets.)

Accounting Beta Measured over:	Stock Beta Measured Over:		
	1946-52	1953-59	1961-68
1946-68 (Excluding 1960)	.32[b] (.22)[a]	.41[b] (.34)[b]	.08 (.0)
1946-52	.29[b] (.05)		
1953-59		.0 (.04)	
1961-68			.07 (.0)

[a] Significant at 5 percent confidence level.

[b] Significant at 1 percent confidence level.

Source: Gonedes [4], Table 5, pp. 434-35.

Gonedes also tested for correlation between the coefficients of determination (R^2's) of the equations used to estimate the accounting and stock betas. The proposition is that if the earnings index explains a large proportion of the variance of a firm's earnings, then the stock market index ought to explain a large proportion of the variance in returns on the firm's stock. Gonedes found this to be true, so long as accounting betas were derived from first differences in accounting income. This is certainly evidence of an association between the cyclicality of a firm's earnings and its stock's market prices. However, correlating R^2's and correlating betas are not the same thing, since a firm can have a low R^2 and high beta, or conversely. Thus it is no paradox to find significant correlation of R^2's but insignificant correlation of accounting and stock betas.

There is no obvious reason for the differences between BKS and Gonedes.[9] Particularly in view of White's results, it seems fair to claim that cyclicality, somehow measured, is a determinant of beta. But the measurement of cyclicality and the exact specification of its relationship to beta still pose difficult problems.

Rosenberg and McKibben

One interesting aspect of the tests described so far is that they have uniformly assumed (1) substantial cross-sectional variance of betas and the real determinants of beta, but

(2) stability of these variables across time for each firm. Assumption (2) is implicit in the way the betas and the independent variables are measured. However, it is obviously questionable. There is no reason to expect the determinants of beta to be strictly constant across time, and if the determinants change, beta ought to change too.

Rosenberg and McKibben [14] use an interesting test which explicitly assumes that beta shifts over time. Space is insufficient for a full presentation, but the idea is this. Suppose beta at time t is linearly related to certain real determinants W_{nt}. For firm j,

$$\beta_{jt} = \sum_{n=1}^{N} W_{jnt} b_j \qquad (3)$$

This relationship is assumed to apply to all firms and to be constant over time. However the real determinants vary across firms and time, and thus so will beta.

Substituting Equation (3) into Equation (1) we have:

$$E(\tilde{R}_{jt}) - R_{Ft} = \sum_{n=1}^{N} W_{jnt} b_j \left(E(\tilde{R}_{mt}) - R_{Ft}\right) \qquad (4)$$

Normally beta is estimated by fitting the regression equation

$$\hat{r}_{jt} = \hat{\alpha}_j + \hat{\beta}_j \tilde{r}_{mt} + \hat{e}_{jt} \qquad (5)$$

where the $\tilde{r}_t$'s are returns in excess of the risk-free rate.

But if Equation (4) is correct, (5) is incorrect, since it assumes the true value of beta is constant. Instead we should run the regression

$$r_{jt} = \alpha_j + \sum_{n=1}^{N} b_j (\tilde{W}_{jnt} \tilde{r}_{mt}) + e_{jt} \qquad (6)$$

where the independent variables are $(\tilde{W}_{jnt} \tilde{r}_{mt})$, the products of the market excess returns and the hypothesized determinants of beta. This procedure allows a direct measurement of the b_j's in Equation (3). This one-step procedure is in contrast to the three studies cited above, in which a two-step test was required— one step to estimate the betas and their hypothesized determinants, and another to see whether the hypothesized variables explain the cross-sectional differences in firms' betas.

The results of most interest here were obtained by pooling time series data (yearly observations) and cross-sectional data for 558 firms over the 1954-66 period. (The data have to be pooled because the b_j's in Equations (3) and (6) are assumed to be the same for all firms.) Unfortunately, the results are hard to compare with those of BKS, White or Gonedes, since Rosenberg and McKibben tried thirty-one independent variables, of which eleven are based on stock market data. Thus, if we find that a variable which we expect to work does not, this may simply be due to its collinearity with one or more of the other 30 variables. There are four direct measures of firm growth, for example, and at least three other variables which might reasonably be expected to proxy for growth. What is one to conclude from the failure or success of any one of these variables?

Moreover, the inclusion of variables derived from stock market data hampers our quest for the real determinants of beta, since the market behavior of the firm's stock responds to the real variables and to some extent proxies for them. In any case, these financial variables are not usually available for specific real assets. (They are for firms, which are collections of assets, but in many instances that is not much help.)

This is actually unfair to Rosenberg and McKibben, who were largely concerned with forecasting betas. Nevertheless, their paper strikes me in the same way that Chinese cooking strikes some people who are brought up on meat, potatoes and gravy. You may be served nine courses, each one delicious, but a half hour after leaving the restaurant you're hungry again. Here we have 31 plausible variables, but a half hour after reading the paper it's hard to say what's been learned about the real determinants of beta.

Of course this is an exageration, because Rosenberg and McKibben do confirm the results of BKS, White and Gonedes in several respects. First, financial leverage is again found to have a significant positive relationship with beta. Second, a positive and highly significant relationship is found for volatility in earnings, measured by the standard deviation of changes in earnings per share over time. Third, strong positive relationships are found for growth in sales and earnings (confirming White and BKS).

On the other hand, no significant relationship is found for either the accounting beta or the dividend payout ratio, which is generally contrary to the other authors I have cited. Whether this is an actual failure of these variables given the Rosenberg-McKibben sample and methodology, or whether it is simply a matter of multicollinearity, is hard to say.

There were several other variables tested. Most were insignificant and some of those that were significant had unexpected signs. Since I could make no sense out of these variables, they will not be reviewed here. Rosenberg and McKibben note that the pattern of signs does not correspond to their *a priori* expectations.[10]

A Tentative Summary

There are several other studies which shed light on the real determinants of beta, but it seems inefficient to lay out the results of each. Instead I will propose a tentative summary at this point, and then note whether the other studies support or weaken this view. It seems safe to identify four factors which generally contribute to high stock betas.

1. Cyclicality. This broad term is meant to include BKS's "accounting beta" and the corresponding risk

measures used in the other studies discussed above. Specific definitions vary, but each study started from the same hypothesis, namely that beta depends on the covariance between swings in the firm's earnings and swings in earnings in the economy generally. This hypothesis is not based on a fully rigorous model, but it is so much in the spirit of the CAPM that it is hard not to hold it *a priori*. The empirical results of BKS and White strongly support the idea and Gonedes's work partly supports it. Rosenberg and McKibben do not find the accounting beta to be significantly related to the stock beta, but I am inclined to think that the effect was stolen by one of their 30 other independent variables.

2. Earnings Variability. Both BKS and Rosenberg-McKibben find earnings volatility to be strongly related to beta. This is mildly disturbing from a theoretical point of view since earnings volatility represents the total, not the systematic risk of earnings; we would expect it to be less important than the "accounting beta" or some other measure of cyclicality. Nevertheless, earnings variability corresponds closely to the popular, intuitive idea of firm risk, and it is sensible proxy for cyclicality. Thus, it certainly belongs on any tentative list of real factors associated with beta.

However, it is unfortunate that no studies have separated the variance of firms' earnings into systematic and unsystematic components, and tested which component is more strongly related to beta. If the CAPM is right, the systematic component ought to be more important.

3. Financial Leverage. Theory specifies a relationship between the market debt ratios and beta, but the effect comes through strongly even when the book debt ratio is used.

4. Growth, which can be measured in a variety of ways. BKS, White and Rosenberg-McKibben all found growth to be important, although the effect seems less strong in the 1960's than earlier. BKS also found dividend payout to be important, which is consistent, since rapid growth normally is associated with low payout, and there is no reason for dividend policy per se to affect risk.[11]

   It is not at all clear how best to measure growth. Moreover, the studies cited have not drawn a clear distinction between growth as expansion and growth defined as the opportunity to invest in projects offering expected rates of return exceeding the cost of capital.

Several other comments should be made before turning to other studies. First, none of the four investigations described above has a complete theoretical base. The authors would of course agree, and note their willingness to use one as soon as it is discovered. But we must bear in mind that the puzzles they encountered may be due to use of the wrong functional specification. Rosenberg and McKibben do at least assume a specific, plausible specification and develop consistent tests, but they would agree that their choice of independent variables is not based on any clear theory.

In the absence of theory there are many intuitively appealing variables and several plausible ways to measure each of them. This naturally makes it difficult to compare the studies cited, except by thinking in terms of general factors like "growth" or "cyclicality."

Other Studies

Several other empirical studies investigate the real determinants of beta. Since there are limits both to space and to the reader's patience, I have chosen not to review them in detail. Instead I will briefly describe whether they support or weaken the case for the four general determinants of beta suggested just above.

Cyclicality. The work of Pettit and Westerfeld [13] and Gordon and Halpern [6] deals primarily with cyclicality as a possible determinant of beta. The former derived an "earnings beta" by first fitting a trend line to firms' earnings per share, scaled by average earnings per share over the period investigated, and also to the scaled earnings of the Standard and Poor's 500 Stock Index. Then the earnings beta was defined as the covariance of the residuals from these trend lines. A highly significant pairwise correlation was found between this statistic and the firms' stock betas. The relationship persisted when the earnings beta was combined with several other independent variables in a multiple regression. Finally, these tests were successfully repeated with a different earnings beta based on operating income.

Gordon and Halpern provide additional evidence as a by-product of their application of the CAPM to the problem of estimating the cost of capital for a division of a firm. Their earnings beta was based on the covariance between the rates of growth of firms' earnings per share and growth rates for economy-wide corporate profits. Again, a strong positive correlation between earnings betas and stock betas was found.

Earnings variability. Lev and Kunitsky [10] find that stock betas are significantly related to the degree of "smoothness" of various operating and financial series, principally earnings, dividends, sales and capital expenditure. Smoothness is measured as the mean absolute deviation of the actual values of the variable from its trend. (If smoothness is S, then a perfectly smooth series will have S = 0.) Clearly this is another way of defining volatility; thus, the discovery of a significant positive relationship between S and the stock beta confirms the results reported earlier in this paper.

On the other hand, it is not obvious why the smoothness of dividends, sales and capital expenditures should matter at all once earnings volatility is accounted for. There are two possible lines of response. The first, and simplest, is to say that any simple measure of earnings volatility is an imperfect representation of investors' actual uncertainty about the firm's future earnings. In this case we can regard the other smoothness measures as useful additional proxies. Lev and Kunitsky see something deeper, however. They argue that firms' managers are engaged in a continual battle to control the environment, and that the greater their success, the lower their firms' stocks' betas. Control of environment is effected by reducing the variance of each of the firm's major activities, not just the variance of the end result, earnings. Thus, smoothness of sales would have a place even if there were no problems in measuring earnings volatility directly. Unfortunately, these explanations are not mutually exclusive, and there is no way to distinguish between them from the results at hand.

The role of earnings volatility is also supported by Lev [9] who shows that firms with high operating leverage (that is, high ratios of fixed to total costs) tend to have high betas. This is sensible since high operating leverage is by definition associated with high earnings volatility and high earnings betas.

Both Pettit and Westerfeld [13] and Melicher [11] confirm that firms with low dividend payout ratios tend to have high betas. This too supports the case of earnings volatility, since low dividend payout is a natural response of a mangement that is uncertain about future earnings.

The only authors who do not find a reliable positive association between earnings volatility and beta are Breen and Lerner [3]. However, their results can be attributed to an inappropriate measure of volatility and to their attempt to measure volatility from severely limited time series data.[12]

Financial Leverage. The debt ratio performs somewhat erratically in the studies of Pettit-Westerfeld, Breen and Lerner, and Melicher. In the first case, the book debt ratio is not significant in a multiple regression with several other independent variables. In Breen and Lerner, the debt ratio is significant but inexplicably of the wrong sign in two out of twelve subsamples. Melicher found it significant only when the square of the debt ratio was also introduced.

These results are not fatal to the theoretical prediction of a positive relationship between the debt ratio and stock beta, other things being equal. You can argue that other things are not equal, so that other variables may be obscuring or proxying for the hypothesized effect. Nevertheless, these results introduce the seed of doubt.

Fortunately for theory, Hamada [8] devined an indirect test which isolates the effect of financial leverage on beta.

There is, of course, substantial variance in the cross sectional distribution of stocks' betas, even for firms in the same industry. Since debt ratios also vary, theory would predict that part of the dispersion is due to different debt policies. Therefore, Hamada grouped a sample of firms into nine reasonably homogenous industries and computed "unlevered betas" for each firm. The unlevered betas were estimates—based on theory, the stock beta and the observed debt ratio—of the beta the firms would have had if their debt ratios were zero. Hamada's prediction was that the dispersion of the firms' betas would be reduced by this unlevering process. (If the stock beta were unrelated to leverage, then the dispersion would be unaffected by unlevering. Since each industry group was chosen to hold other determinents of beta roughly constant, the unlevered beta estimates should cluster more closely around some true "industry beta.") Hamada's prediction is confirmed by his results, which justify substantial confidence in the link between financial leverage and the stock beta.

Growth. Growth is the weakest of the four candidates which I tentatively proposed as real determinants of beta. Its weaknesses show up again in Pettit and Westerfeld's study. Although they find a significant pairwise correlation between the growth rate of earnings per share and the stock beta, the variable is insignificant in multiple regressions. Breen and Lerner find a generally significant relationship, but again it has the wrong sign (negative) in two of the twelve subsamples examined.

On the other hand, the dividend payout ratio is negative and significant in these two studies, as well as in Melicher's. Perhaps this ratio is simply a better proxy for the firm's growth prospects. Unfortunately, none of the studies cited in this paper have attempted to determine what the dividend payout ratio is really proxying for, or whether it has an independent effect on beta.

Summary

The empirical evidence on the real determinants of beta can be summed up in two sentences. At least three real factors can be identified with relative confidence, namely financial leverage, cyclicality and volatility of operating earnings (although the third may be proxying for the second). Growth is a possible fourth variable, but its performance in the empirical work cited has been erratic.

The summary has to be brief, because there is very little to say once the four factors are noted. despite the substantial number of studies cited, there is scant agreement on how the factors are to be measured and the exact specification of their effects on beta. Theory gives no guidance (except in the case of financial leverage), so there is little to say beyond the generalities in the paragraph just above. Further progress in understanding the real determinants of beta depends on the development of a theory which specifies the relevant variables and how they should in principle be measured. A modest beginning is made in the next section.

FOOTNOTES

FOR S. C. MYERS' ARTICLE

1. The best-known proponent of this view is Gordon [5].

2. Hamada [8].

3. Dyer has found evidence of this effect. See J. Dyer, "Financial Leverage, Business Risk and the Tax Subsidy to debt: An Empirical Study." Unpublished M.Sc. Thesis, MIT, 1974.

4. Errors in observing true values of beta and the accounting variables ought to be reduced by forming portfolios.

5. The explanatory power was only slightly less when the accounting beta was substituted for earnings volatility.

6. Actually, White used the ratio of the book value of debt to the market value of equity. It is the divergence between book and market values of equity which causes most of the error in the book debt ratio.

7. And also for various portfolio sizes, variable definitions, etc.

8. Beaver, Kettler and Scholes [2], p. 667.

9. It might be noted that Gonedes's sample was smaller than that of BKS or White, and moreover that his sample was concentrated in a few specific industries. Gonedes

properly criticizes BKS for normalizing the earnings of firms in their sample by past market values of equity. Firms with high betas will tend to have more volatile equity values, and under BKS's procedure, will automatically tend to have more volatile earnings and higher accounting betas. Thus BKS are not correlating beta solely with accounting variables, but also with stock price volatility.

10. Rosenberg and McKibbin [14], p. 327.

11. This follows from Miller and Modigliani's well-known proof of the irrelevance of dividend policy [12].

12. They first fitted a trend line to five years' earnings per share data, and then measured volatility as the proportion of the variance explained by the trend. This is a poor procedure, not only because of the small number of earnings observations, but because there is no necessary relationship between R^2 and the standard deviation of the series. A firm's earnings could have a low standard deviation but no trend and an R^2 of essentially zero. Breen and Lerner would take the low R^2 as evidence of high volatility in earnings.

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S. C. MYERS' ARTICLE

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11. Melicher, R. W., "Financial Factors Which Influence Beta Variations Within a Homogeneous Industry Environment," *Journal of Financial and Quantitative Analysis*, IX (March 1974), 231-43.

12. Miller, M. H. and F. Modigliani, "Dividend Policy, Growth and the Valuation of Shares," *Journal of Business*, 34 (October 1961), 411-53.

13. Pettit, R. and R. Westerfeld, "A Model of Capital Asset Risk," *Journal of Financial and Quantitative Analysis*, 7 (March 1972), 1649-68.

14. Rosenberg, B. and W. McKibben, "The Prediction of Systematic and Specific Risk in Common Stocks," *Journal of Business and Quantitative Analysis*, VIII (March 1973), 317-34.

15. White, R., "On the Measurement of Systematic Risk," Unpublished Ph.D. Dissertation, MIT, 1972.

(End of Excerpt from S. C. Myers' Article.)

(b) Recent Studies

Two new studies have been published since the Myers survey. The first is by Beaver and Manegold, the second by Gonedes.

Beaver and Manegold

The Beaver and Manegold Study [1] concentrates exclusively on the relationship between market and accounting betas.

The market betas were computed in the usual manner by regressing monthly stock returns on the corresponding returns for the NYSE index. Three accounting betas were computed for each stock using different definitions of accounting return. (Note that the lack of a theoretical model leaves the specification of the accounting return unclear.) The returns used were: (1) net income to total assets, (2) earnings to net worth, and (3) earnings to market value of net work (shares x price). These returns were computed for each sample firm (254 companies) for each test year (1951-69). The first two return measures are based solely on book data, the third combines book and market data. For each year, the returns on the index portfolios were unweighted averages of the 254 stock returns.

The data base for the study consisted of 254 firms (all those on the Compustat Annual Industrial Tape with complete data over the 1951-69 test period). Betas were computed for the total period and for two subperiods (1951-60, 1961-69). In addition to the procedures summarized here, several other refinements were employed to reduce sampling errors in the estimated market and accounting betas (for example, a Bayesian correction to the market betas; also accounting betas were produced using first differences of accounting returns).

The end result of these various testing refinements was 378 sets of market and accounting betas. A correlation coefficient was measured for each set. Of the 378 total, 363 of the coefficients were statistically significant beyond the 0.01 level, while 375 (all but three) were significant beyond the 0.05 level. The implications of this finding are that across a wide variety of specifications for accounting betas and market betas, there is a statistically significant relationship between the two. Alternatively stated, there appears to be a significant relationship between risk as reflected in security prices and risk as reflected in accounting earnings data, and the existence of the relationship can be detected in a variety of forms.

Apart from the issue of statistical significance is the magnitude of the correlation coefficients. For the total period, cross-sectional differences in the 254 accounting betas explained only about 15 to 20 percent of the cross section variation in market betas. For the first period, the proportion explained is approximately 20 to 25 percent. For the second period the account betas based on the first two accounting return measures explain only 3 to 6 percent and the third form of the accounting betas explains about 25 percent.

A sample scattergram (one of 378 possible) is shown in Figure 3-4. The figure shows the relationship between total period market betas (vertical axis) versus total period accounting betas. The accounting betas are based on the net income to total assets return measure. The fact that a relationship exists between the market and accounting betas is evident from Figure 3-4. It is also evident, however, that accounting betas do not explain much of the variation in market betas.

The small proportion of variation accounted for in the Beaver Manegold Study is due to a combination of two reasons. First, the accounting betas have substantial measurement errors (high standard errors) due to the small number of annual observations available (compared with the market betas which are based on monthly data). Second, the accounting beta is only one of several possible factors that determine a securities systematic risk. While both factors are obviously involved, it is not possible to determine their relative importance in this study.

Gonedes

A second article by Gonedes [illegible] published in June 1975 [5] updating his earlier results [4]. The topic again was the relationship between market and accounting betas.

The model used to obtain market betas was the familiar market model

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \varepsilon_{it} \tag{3-6}$$

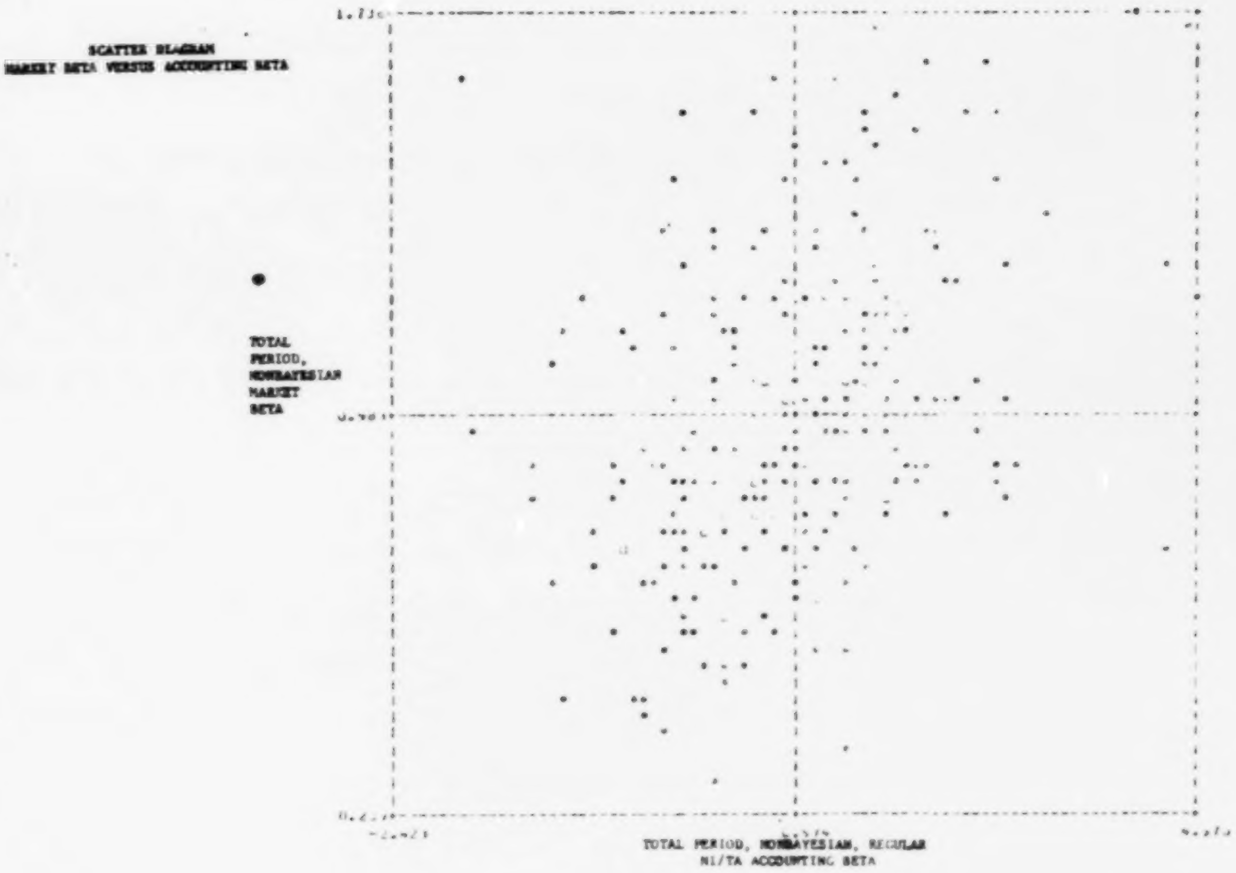

Figure 3-4. Scatter diagram: market beta versus accounting beta. (Source: Beaver, W. and Manegold, J., [1], Figure 5, page 259.)

where

$\tilde{R}_{it}$ = the rate of return on security i during period t (tilde ($\sim$) denotes random variable)

$\tilde{R}_{mt}$ = the rate of return on the market portfolio in period t

An analogous model was used to measure the accounting betas (designated δ in Equation (3-7)):

$$\tilde{V}_{it} = \gamma_i + \delta_i \tilde{V}_{mt} + e_{it} \qquad (3\text{-}7)$$

where

$\tilde{V}_{it}$ = the rate of return, based on accounting numbers, for firm i in period t

$\tilde{V}_{mt}$ = the market-wide index of rates of return, based on accounting numbers

The specific measure of accounting return used by Gonedes was the ratio of net income (for year t) to the book value of common equity at the beginning of year t. The market-wide index of returns used was a common-equity-weighted index of the accounting returns for the firms in the sample. (The method of index construction was one of the principal differences between the two Gonedes studies.)

In addition, accounting betas were produced using first differences of the stock and index accounting returns.

The data sample contained 316 firms (as opposed to 99 in the first study). The accounting betas were computed from annual data for the period 1946-69 (excluding 1953, 1960, and 1968, which were reserved for various predictive tests not described here). The market betas were computed using monthly data for three intervals, 1946-52, 1953-60, and 1961-68. The 316 accounting betas were then correlated with the market betas from each of the subperiods. The results were somewhat more encouraging than in the first study. The accounting betas produced from Equation (3-7) tend to be significantly related to the market betas, which was not the case in the earlier study. Further, the difference in the results for the ordinary and first difference accounting returns is now almost negligible (which is more consistent with the findings of the other studies). However, Gonedes still finds the proportion of the variation in market betas explained by the accounting betas to be smaller than in other studies. For example, the proportion of cross-sectional variation explained in the various Gonedes regressions ranged from only 1 to 2 percent——very small indeed.

(c) Summary

The research into the nature of the real determinants of beta is almost entirely of an empirical nature. Much of the confusion that exists with respect to the measurement of variables, the specification of tests, and the interpretation of empirical findings results directly from the lack of an acceptable theoretical framework. The empirical studies have identified four factors which seem to be systematically related to beta: cyclicality and variability of earnings, leverage, and growth.

i. Cyclicality of Earnings

Cyclicality is the tendency for corporate earnings to move with earnings in the economy generally. Earnings have been measured in different ways in the various studies. However, the accounting return on equity is commonly used (earnings divided by book value of equity). Cyclicality is measured by an accounting beta, computed by regressing these returns on market-wide index of accounting returns (see Equation (3-7)). The accounting betas are typically found to be related to the market betas in a statistically significant manner. However, the authors disagree on the proportion of the cross-sectional variation in market betas explained by the accounting betas; percentages range from 1 percent to 40 percent, depending on the sample of firms and time periods involved. A best guess would put the central tendency between 10 and 20 percent.

ii. Earnings Variability

Earnings variability is measured by the standard deviation of historical earnings. It is a measure of the total risk of book earnings, while cyclicality is a measure of systematic earnings risk. Several studies have found variability as well as cyclicality to be strongly related to the market beta. This result is somewhat surprising, since portfolio theory would predict that only the systematic component of earnings risk and not the total should be important. It may well be, however, that the variability is serving as a proxy for cyclicality. It is hoped that future studies will more carefully segregate the systematic and nonsystematic components of book earnings risk.

iii. Leverage

Financial leverage (the ratio of debt to total assets) shows up significantly in almost all the studies where it is considered. This is true whether the debt ratio is defined in book (i.e., accounting) or market value terms. Leverage is the one variable for which a specific theoretical relationship exists. As given by Hamada [6],

$$\beta_L = \beta_0 \left[1 + (1 - T_c) \frac{D}{E} \right] \qquad (3\text{-}8)$$

where

β_L = the market beta for the stock of a levered firm

β_0 = the unlevered beta (i.e., the beta of the stock if the firm was all-equity financed)

$\frac{D}{E}$ = the market value debt-to-equity ratio

Thus, for firms with similar business risks (as reflected by β_0), the use of leverage will directly increase the risk of the common stock (as given by β_L). This relationship, while difficult to test directly, has been tested in indirect ways. The results support the hypothesis (see Hamada [7]).

iv. Growth

Growth is measured in various ways in the different studies (e.g., growth in earnings or sales) and captured by various proxies (e.g., dividend payout ratio). The growth variables were the most erratic of these four determinants. It is commonly held that high-growth firms are riskier (i.e., they have higher market betas). This hypothesis was not conclusively borne out in the studies reviewed. In some of the multiple regressions growth was significant, but sometimes the relationship was the reverse to that predicted.

FOOTNOTES FOR PART 3

1. The summary of risk-return concepts contained in Section II is based on a presentation by Stewart C. Myers and Gerals A. Pogue [20], pp. A3-A20.

ESTIMATION OF THE COST OF CAPITAL FOR MAJOR UNITED STATES INDUSTRIES WITH APPLICATION TO POLLUTION-CONTROL INVESTMENTS

PART 4. ESTIMATION

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November 1975

I. INTRODUCTION [1]

Part 4 contains the estimation methodology and empirical results. It is divided into seven sections.

Section II shows how the discounted cash flow cost of capital models and the risk-return concepts of Parts 2 and 3, respectively, can be combined to produce cross-sectional estimation equations. Section III describes the primary and secondary company samples used in the study. Section IV presents the equity risk measures for sample firms. Section V describes the procedures used to estimate the cross-sectional regression variables and presents the cost of equity capital estimates. Section VI contains the methodology used to estimate weighted average costs of capital and presents the results. Finally, Section VII shows how the weighted costs are likely to change during the 1975-84 period given estimates of future interest rates.

II. METHODOLOGY

(a) Combining the Cost of Capital and Risk-Return Models

In Part 2 three security valuation models were presented which permitted estimates of a firm's cost of equity capital. The models differ only in their assumptions regarding the firm's future investment opportunities. The models are:

1. Perpetual Growth Model

$$R = \frac{D_1}{P_0} + g \qquad (2\text{-}8)$$

2. No (Real) Growth Model

$$R = \frac{E_1}{P_0} \qquad (2\text{-}10)$$

3. Finite-Period (Real) Growth Model

$$R = \frac{E_1}{P_0} + \frac{I_1}{P_0}(r - R)T \qquad (2\text{-}12)$$

where the variables in the three models are collectively defined below:

R = the cost of equity capital

P_0 = the current share price of the firm's common stock

E_1 = the expected earnings per share during the next year

D_1 = the expected dividends per share during the next year

I_1 = the expected equity investment per share during the next year (i.e., retained earnings)

g = the expected growth rate of earnings and dividends

r = the rate of return on reinvested earnings

T = the number of years the firm is expected to have "real" growth opportunities (i.e., $r \neq R$)

These models are typically used to produce estimates of the cost of equity for individual firms. However, the market's expectations for the various variables required by the models cannot be precisely estimated using simply historical data. Thus, the models will estimate equity costs with error. But, as described in Part 2, the combined estimates of models 1 and 2, and the estimates of model 3 should be reasonably unbiased (if noisy) proxies for the true costs.

If all of the firms in the data sample had the same equity risk, it would be a straightforward matter to estimate the cost of equity capital. The optimal estimate would be a weighted average of the individual values; individual company estimates with lower measurement error would be weighted more heavily in the average cost figure.

Unfortunately, the samples do not have the same risk. Sample industries cover a wide range of equity risks. Even within industries the companies differ significantly in risk. What is needed is an estimation procedure that explicitly allows for differences in company risk. The risk-return theory described in Part 3 provides a basis for such a procedure. The Capital Asset Pricing Model (CAPM) specifies a relationship between capital costs and equity risk, as measured by beta:

$$R = R_F + \beta(R_m - R_F) \qquad (3\text{-}3)$$

where

R = the cost of equity capital (i.e., the required or expected rate of return on equity)

β = the beta value for the firm's stock

R_m = the expected rate of return on the market

R_F = the risk-free rate

A generalization of this model (the two-factor model) was described in Part 3 as well (see Equation (3-4)). Both models hypothesize a linear relationship between the cost of capital (R) and risk (β).

It is not assumed in this study that the CAPM is empirically valid. The tests of the model summarized in Part 2 indicate that the slope of the actual risk-return line tends to be flatter than that predicted by the CAPM. However, the linearity of the risk-return relationship has been strongly established in several studies. In this research this result is used. That is, the relationship between the cost of capital and beta is assumed to be given by

$$R = a + b(\beta) \qquad (4\text{-}1)$$

where a and b are constants. Comparison of Equations (3-3) and (4-1) shows that if the CAPM was empirically valid, then the estimated values of a and b should approximate the risk-free rate (R_F) and the market risk premium (R_m - R_F) respectively.

(b) Estimation Equation (1-3)

The risk-return relationship (Equation (4-1)) provides no insight as to how R should be estimated. Procedures for estimating R for individual companies are given by the three cost-of-capital models. Thus, combining Equation (4-1) with the three models will result in estimation equations in which all of the variables are directly observable or estimable. Each of these equations will be dealt with in turn.

1. Perpetual-Growth Estimation Equation (Model 1)

$$\left(\frac{D_1}{P_0} + g\right)_j = a + b(\beta_j) + \epsilon_j \qquad (4\text{-}2)$$

$$j = 1, \ldots, n$$

The j subscript designates the company. The regression coefficients are estimated cross sectionally using "point-in-time" estimates for the n sample firms.

The ϵ_j term represents the measurement error resulting from the use of Equation (2-8) to estimate R_j. The ϵ_j is the difference between the true (unobservable) cost of capital and the estimated value from Equation (2-8).[2]

Since the variables in Equation (4-2) are estimates of the "true" values, it is common practice to put hats (^) over them to so indicate (e.g., β_j). This has not been done, however, for reasons of notational simplicity.

2. No-Growth Estimation Equation (Model 2)

$$\left(\frac{E_1}{P_0}\right)_j = a + b(\beta_j) + \varepsilon_j \qquad (4-3)$$

$$j = 1, \ldots, N$$

For Models 1 and 2, the estimated cost of capital for firm j is given by $\hat{a} + \hat{b}(\hat{\beta}_j)$ where $\hat{a}$ and $\hat{b}$ are the estimated values for a and b resulting from cross-sectional regression analysis for a specified date (e.g., December 1974). (The cross-sectional procedures are described in subsection (d).)

3. Finite-Growth Estimation Equation (Model 3)

The finite-growth model is more complicated and several difficulties must be resolved before it can be used to estimate R. These involve the T parameter and the recursive nature of Equation (2-12).

The T parameter measures the duration of real growth. It is not directly observable and must be estimated from the regression analysis. Conceivably, every sample firm would have a different T value. However, this would prohibit estimation of any T values. Another assumption would be to assume all firms have the same T value. But this would be unrealistic. A compromise assumption which has considerable merit is to assume the T is reasonably constant for firms in an industry group. The data base of the study consists of eight primary sample groups (see Section III for a description of the data base used). Thus, a duration parameter is defined for each of the eight industries (T_k, $k = 1, \ldots, 8$). Since the duration variables are not directly observable (i.e., they must be estimated from the regression analysis), they are replaced by regression coefficients (C_k, $k = 1, \ldots, 8$). These coefficients will be estimated from the cross-sectional regression along with a and b.

Each firm belongs to only one industry. To separate the firms into groups, a series of eight "dummy" variables (Z_k, $k = 1, \ldots, 8$) are defined. For each sample firm only one of the Z_k variables will equal 1, the remainder will be zero. The form of the estimation equation is given by

$$\left(\frac{E_1}{P_0}\right)_j = a + b(\beta_j) + \sum_{k=1}^{8} C_k \left[(r-R)\,\frac{I_1}{P_0}\right]_j Z_k + \varepsilon_j$$

$$j = 1, \ldots, N \qquad (4\text{-}4)$$

Note that Equation (4-4) has nine independent variables: β_j and eight growth terms. For each firm only the growth term corresponding to its group membership will be non-zero.

The next problem with the finite-growth model is its recursive nature. The term recursive refers to the fact that R in Equation (2-12) is defined in terms of itself. As a result, the growth terms in Equation (4-4) contain R. While the other variables in the growth terms (P_0, r, I_1) can be either observed or estimated prior to running the regression analysis, R obviously cannot be; R is the object of the whole exercise.

The solution is to perform the regression analysis for Equation (4-4) on an iterative basis. That is, initialize R in Equation (4-4) with a best guess, estimate the a, b, and C_k ($k = 1, \ldots, 8$) parameters, re-estimate R using Equation (4-4) (i.e., $R = \hat{a} + \hat{b}(\hat{\beta}_j)$) and proceed as before. The iterative process will continue until the estimated parameters stabilize.

The next question is how to initialize the process; what should be the first "best guess" for the R_j values? The obvious answer is to initialize with the perpetual-growth (Equation (2-8)) or no-growth (Equation (2-10)) values. If the iteration process is valid, it should not depend on which estimate is used for initialization. This is the procedure used in this study.

(c) Model 4—The Long-Run Risk-Return Line

It would be possible to estimate the cost of equity capital directly from the Capital Asset Pricing Model (CAPM). According to the model (see Equation (3-3)), the rate of return expected by stockholders is the sum of the risk-free rate (R_f) plus a risk premium that is proportional to the stock's beta. However, strict application of this formula poses several difficulties. First, the long-run relationship between risk and return is flatter than the CAPM predicts. Second, what is the appropriate risk-free rate? Should we use the rate on short-term or long-run government bonds? Third, what returns do investors expect on the market—i.e., what is R_m?

In view of these problems and particularly the first, it would be unwise to rely solely on the CAPM to estimate R. Nevertheless, it would seem useful to consider the implications of the long-run relationship between realized stock returns and risk.

The fourth estimation equation is an extended version of the CAPM which permits a flatter risk-return line than the original model (and thus eliminating the first difficulty with the CAPM). It is given by

$$R_j = R_F + \beta_j(R_m - R_F) - (\beta_j - 1)(1 - \delta)(R_m - R_F)$$

Rearranging terms, we obtain

$$R_j = [R_F + (R_m - R_F)(1 - \delta)] + \beta_j(R_m - R_F)\cdot\delta \tag{4-5}$$

where δ is the ratio of the actual to the CAPM-predicted slope of the long-run risk-return line. δ was estimated as follows. The slope of the relationship between return and risk for all stocks listed on the New York Stock Exchange between March 1931 and June 1970 was 0.827 percent per month. The predicted value from the CAPM was 1.171 percent per month. The slope attenuation function, δ, is the ratio of these two numbers, or 0.706. In its predictions of R_j, this extended model will use a relationship between return and risk which is consistent with long-run experience, as opposed to the one predicted by the CAPM.

The second and third difficulties with the CAPM remain; that is, what values should we use for R_F and R_m? Unfortunately, no complete solution exists, and one can only try to do what seems reasonable. For R_F the 1-year government rate for the 12 months following the estimation date was used. The expected market return, R_m, was estimated by applying the perpetual-growth model, Equation (2-8), to the market index. The index used was the Standard and Poor's 500 Stock Composite Index. Since the index represents the market as a whole, there is no *a priori* reason to believe that Equation (2-8) would produce a seriously biased estimate of R_m. (The values used for the 1971-74 estimation equation are given in Table 4-8.)

(d) Cross-Sectional Regression Procedures

Cross-sectional regression analysis was used to estimate the coefficients of Equations (4-2) through (4-4) for four dates—December 1971 through December 1974. This procedure pools the data from the N sample firms at each date to estimate the relationship between equity cost and risk existing at that time.

Prior to performing the cross-sectional regressions, preliminary time series studies were performed for each firm to estimate the necessary dependent and independent variables (e.g., $\hat{\beta}_j$, $\hat{g}_j$). The details of the time series variable estimation are described in Sections IV and V.

Weighted Regression Procedures

The three discounted cash flow (DCF) cost-of-capital models are, in effect, used to prepare preliminary estimates of the equity cost of each sample firm. These estimates are then combined with the beta measures in cross-sectional Equations (4-2) through (4-4) to produce new estimates for R_j

(i.e., $\hat{R}_j = \hat{a} + \hat{b}(\hat{\beta}_j)$). This procedure combines the information in all N observations to produce more efficient estimates of $\hat{R}_j$ than can be obtained individually by the DCF models. This is the rational for the cross-sectional procedure.

The estimates of R_j from the DCF models contain different amounts of information. For some companies the historical data permit estimates of R_j having relatively little uncertainty (i.e., small measurement error). For others the error will be so large that estimates of R_j will convey little information about the true value.

Thus, when estimating the risk-return relationship at a given time (i.e., $\hat{a}$, $\hat{b}$, and $\hat{C}_k$ for December 1974), it would not be useful to give each DCF $\hat{R}_j$ equal weight in the cross-sectional regression. It is preferable from the statistical point of view to weight each company estimate by its information content.

The appropriate weighting factors are related to the standard deviations of the measurement errors of the DCF estimates. The larger the standard deviation, the smaller the amount of information about the true R_j conveyed by the estimate. Thus, the weighting factors would be the reciprocal of the standard errors. Each of the three cross-sectional equations would thus use different (but obvious highly related) sets of weights.

In this research the weights used for all three cross-sectional equations are the reciprocals of the standard error of past earnings growth. This weight is used as a proxy for the weight described above which is not observable.

The standard error of past earnings growth is hypothesized to be a good proxy for the unobservable measurement error standard deviation. The degree of confidence we can have in individual DCF R_j estimates depends on our ability to infer the future from historical data. In some cases, where past trends are well defined, we can feel somewhat confidence regarding our estimates. However, where the historical pattern is murky, the time-series estimates will tend to be unreliable predictors of what the market expects.

Further, since the valuation equations rely on the same set of historical data for estimation, the measurement errors of the R_j estimates produced by Equations (2-8), (2-10), and (2-12) will tend to be of comparable size. Thus, as an approximation, it is not unreasonable to consider the same set of weights to be applicable for all three DCF estimates. In this study the same set of weights has been used for all three cross-sectional regressions.

Details on estimation of the weights are contained in Section V(b).

Weighted Cross-Section Estimates of R_j

Estimates of R_j based on the cross-sectional results were prepared for each of the three estimation equations as follows:

$$\hat{R}_j^{\ell} = \hat{a}^{\ell} + \hat{b}^{\ell}(\hat{\beta}_j) \qquad \ell = 1, 2, 3 \qquad (4\text{-}6)$$

where the $\hat{a}$ and $\hat{b}$ are regression estimates for a and b, and the ℓ superscript indicates the model number ($\ell = 1, 2, 3$). Additionally, for Model 3 the $\hat{C}_k$ estimates indicate the duration of real growth for each of the eight industry groups. Details of the results are left to Section V.

(e) Combining the Results of Models 1 through 4

The next step is to combine the four estimates into a single estimate. This was done by average the four values. An unweighted average was used since no compelling reason existed for weighting the results of one model more than another.[3] The combined estimate, R_j, is given by[4]

$$\hat{R}_j = \frac{1}{4}\left[\hat{R}_j^1 + \hat{R}_j^2 + \hat{R}_j^3 + \hat{R}_j^4\right] \qquad (4\text{-}7)$$

$$j = 1, \ldots, N$$

It now remains to estimate an error range for the combined equity cost estimates, $\hat{R}_j$. This is accomplished by first examining the relationship between the cost of equity capital and beta produced when the four models are combined. That is,

$$\hat{R}_j = \hat{a} + \hat{b}(\beta_j) + \varepsilon_j \qquad (4\text{-}8)$$

where the $\hat{a}$ and $\hat{b}$ terms are unweighted averages of the values from the four equations (see Table 4-9 for estimates of $\hat{a}$ and $\hat{b}$). Now, the uncertainty in the estimated cost derives from the variation in the four estimates, plus the standard error of the beta coefficients (SE·β). The standard error associated with the equity cost estimates, $\hat{\sigma}_j$, is measured by

$$\hat{\sigma}_j = \left[[(SE \cdot \beta)\ (\hat{b})]^2 + s_j^2 \right]^{\frac{1}{2}} \qquad (4\text{-}9)$$

where s_j^2, the variance of the ε_j variable, was estimated from the variance of the $\hat{R}_j^{\ell}$ estimates ($\ell = 1, 2, 3, 4$).

III. DEFINITION OF THE PRIMARY AND SECONDARY SAMPLES

The primary purpose of the study was to estimate the costs of capital for six basic American industries: pulp and paper, chemicals, petroleum refining, iron and steel, non-farrous metals, and utilities.

The data base for the study was obtained from the Compustat PDE tape.[5] This tape contained 314 firms in the six groups listed above. The data tape is organized along industry lines as defined by the Standard Industry Codes (SIC). The six groups are spanned by eight two-digit SIC code groups. The distribution of firms in these two-digit industries is shown in Table 4-1.

The 314 firms were divided into primary and secondary samples.

The primary sample consists of all companies with sufficient historical data on the tape to permit inclusion in the equity cross-sectional regressions. To be included, a company must have at least 7 years of earnings data on the tape. The primary sample included 208 of the 314 companies. The distribution by group is shown in Table 4-1.

The secondary sample is a further classification of the 314 firms into the original six groups plus 26 subgroups. (The subgroup names are given in Table 4-3.) The subgroups represent an attempt to build smaller homogeneous groups for the purpose of later analysis. The major stratifying factors were: size of operation (petroleum refining), degree of resource control (papers), product similarities (chemicals, steel, non-ferrous metals), and accounting convention (utilities). The secondary sample included 207 firms. A high degree of overlap existed between the two samples, but several firms in the secondary sample were not contained in the primary sample. For these firms, cost of equity capital estimates were imputed from the results of the primary sample by substituting their estimated betas into Equation (4-8)

TABLE 4-1

DEFINITION OF SAMPLE SIZES

NUMBER OF COMPANIES

GROUP NAME	SIC CODES	TOTAL ON TAPE	PRIMARY SAMPLE	SECONDARY SAMPLE
MINING *	1000-1031	35	18	6
FOREST PRODUCTS **	2400	21	9	5
PAPER **	2600-2650	49	31	49
CHEMICALS	2801-2803	54	36	36 ***
REFINING	2911-2913	43	34	36
STEEL	3310-3317	50	28	27 ***
NON-FERROUS *	3331-3350	27	17	13
UTILITIES	4911-4912	35	35	35 ***
TOTAL		314	208	207

* Combined as Non-Ferrous Metals group in secondary sample.

** Combined as Pulp and Paper in secondary sample.

*** All companies in secondary sample are also in primary sample.

IV. MEASUREMENT OF EQUITY RISK

(a) Calculation of Betas

The basic data for estimating common-stock betas are monthly rates of return during the February 1962 through December 1974 period. The stock returns were computed from the Compustat PDE tape.

The beta for a security is calculated by regressing the monthly security risk premiums on the observed risk premiums for the market. The risk premiums are formed by subtracting the 30-day treasury bill rate from both the stock and market returns. It is customary to convert the rates of return to risk premiums to remove a source of "noise" from the return data. The noise stems from the fact that observed returns may be higher in some years simply because the risk-free rates of interest are higher. Thus, an observed rate of return of eight percent might be regarded as satisfactory in 1960, but as a relatively low rate of return when interest rates were at all-time highs in 1969. The form of the estimation equation is given by:

$$r_t = \hat{a} + \hat{\beta} r_{mt} + \varepsilon \qquad (4\text{-}10)$$

$$t = 1, \ldots, 155$$

where r_t and r_{mt} are the observed risk premiums on the stock and market index respectively during month t.

Betas were estimated for all stocks in the primary and secondary samples. The market index used was the Standard and Poor's 500 Stock Composite Index (with dividends reinvested). The risk-free rate used to convert returns to risk premiums was the 30-day treasury bill series.

The data tape began in February 1962 and ended in December 1974. For stocks with complete histories on the tape, the full period was used in computing beta (155 months of returns). For non-complete companies, all available data was used. All companies in the primary sample had at least 5 years of monthly return data.

Betas for groups and subgroups were computed by averaging the betas for the companies in the group. This results in the same beta as if the returns were first pooled into a group portfolio and the portfolio beta computed as described above.

Beta Results

The average betas and standard errors of beta for the eight primary groups are shown in Table 4-2 (first two numerical columns). In addition, the standard error of the mean beta is shown in parentheses under the mean. The data show forest product common stocks to be the most risky (mean beta = 1.37) and, as expected, utilities the least risky (mean beta = 0.70). When comparing mean betas, the reader must keep the standard errors in mind. Differences in sample means can only be considered statistically significant if large relative to the standard errors of the means.

Similar results for the 26 secondary sample subgroups are given in Table 4-3 (first two numerical columns). The highest subgroup beta is 1.75 for the smallest category of integrated petroleum refiners (10 - 30 thousand barrels per day). However, this group contains only one company, and the standard error of the measured beta is large (0.26). Hence, this result is suspect. The lowest risk subgroup is flow through earnings public utilities with mean beta equal to 0.68.

TABLE 4-2

STOCK RISK AND CAPITAL STRUCTURE DATA
PRIMARY GROUP AVERAGES

GROUP NAME	STOCK RISK		MARKET CAPITALIZATION			
	AVG. BETA	AVG. SE · β	% DEBT	% LEASE	% PREF	% COM.
MINING (18)	0.94 (0.05)	0.18	10.3	3.1	1.2	85.4
FOREST PRODUCTS (9)	1.37 (0.08)	0.18	22.0	11.1	2.0	65.0
PAPER (31)	0.97 (0.05)	0.17	20.6	13.1	2.7	63.6
CHEMICALS (36)	1.22 (0.06)	0.17	19.9	17.2	2.6	60.4
REFINING (34)	1.06 (0.06)	0.17	14.6	19.4	2.6	63.4
STEEL (28)	1.00 (0.05)	0.17	25.5	10.1	3.1	61.3
NON-FERROUS (17)	1.07 (0.07)	0.17	24.1	11.2	0.7	64.4
UTILITIES (35)	0.70 (0.01)	0.09	44.1	0.0	9.6	46.3
TOTAL (208)	1.01 (0.02)	0.16	23.6	11.1	3.6	61.7

( ) = Standard error of mean beta.

TABLE 4-3

STOCK RISK AND CAPITAL STRUCTURE DATA

SECONDARY GROUP AVERAGES

GROUP	SUBGROUP		STOCK RISK		MARKET CAPITALIZATION			
			AVG BETA	AVG SE-β	% DEBT	% LEASE	% PREF	% COMMON
PULP AND PAPER	1	LARGE MULTIPRODUCT COMPANIES (8)	1.03 (0.06)	0.13	22.7	11.7	2.6	62.9
	2	MEDIUM WITH RESOURCE CONTROL (19)	0.87 (0.06)	0.17	22.1	13.9	3.3	60.7
	3	SMALL-MEDIUM W/O RESOURCE CONTROL (10)	0.73 (0.15)	0.29	17.5	13.6	2.1	66.9
	4	FOREST PRODUCTS COMPANIES (5)	1.20 (0.05)	0.23	20.7	7.2	2.9	69.1
	5	CONVERTERS (12)	1.13 (0.10)	0.31	19.1	20.2	2.3	58.4
CHEMICALS	6	CHEMICALS—MAJOR (11)	1.12 (0.04)	0.11	18.1	14.8	1.3	65.8
	7	CHEMICALS—INTERMED. (6)	1.23 (0.08)	0.10	21.8	18.9	6.1	53.2
	8	CHEMICALS—SPECIALTY (19)	1.28 (0.10)	0.19	20.3	17.9	2.2	59.6
PETROLEUM REFINING	9	INTEG. MAJORS (14) (200 + B/D)	0.91 (0.04)	0.11	14.2	24.0	3.2	58.6
	10	INTEG. LARGE (5) (200 + B/D)	0.67 (0.06)	0.17	14.0	22.7	4.2	59.1
	11	INTEG. (3) (100 - 200 B/D)	1.31 (0.06)	0.18	16.7	24.6	1.4	57.3
	12	REFINERS (1) (100 - 200 B/D)	0.96 (NA)	0.25	32.2	7.2	5.5	55.1
	13	INTEG. (2) (70 - 100 B/D)	1.69 (NA)	0.36	25.3	41.2	3.6	29.9
	14	INTEG. (2) (30 - 70 B/D)	1.33 (NA)	0.27	29.0	11.8	0.6	58.6
	15	INTEG. CAN. (2) (30 - 70 B/D)	0.79 (NA)	0.24	31.2	14.0	12.5	42.3
	16	REFINERS (2) (30 - 70 B/D)	1.24 (NA)	0.33	19.4	18.0	0.0	62.6
	17	REFINERS (3) (10 - 30 B/D)	1.68 (0.26)	0.31	13.4	12.2	2.7	71.7
	18	INTEG. (1) (10 - 30 B/D)	1.75 (NA)	0.26	11.0	5.9	6.8	76.3
IRON AND STEEL	19	STEEL—MAJOR (7)	0.94 (0.05)	0.13	30.8	10.3	2.7	56.1
	20	STEEL—MINOR (20)	1.02 (0.06)	0.18	24.3	10.0	3.4	62.3
NON-FERROUS METALS	21	PRIMARY COPPER (8)	1.12 (0.07)	0.15	16.8	3.8	0.8	78.6
	22	PRIMARY LEAD AND ZINC (3)	1.22 (0.32)	0.18	25.4	4.6	6.3	63.6
	23	PRIMARY ALUMINIUM (4)	1.15 (0.13)	0.14	40.4	16.1	2.8	40.7
	24	SECONDARY SMELTING (4)	1.10 (0.11)	0.26	32.6	16.9	1.3	49.1
UTILITIES	25	UTILITIES FLOW THRU (15)	0.68 (0.03)	0.09	45.5	0.0	10.3	44.2
	26	UTILITIES NORMALIZED (16)	0.72 (0.04)	0.09	42.5	0.0	8.8	48.7

( ) = Standard error of mean beta.

NA = Not Available.

V. COST OF EQUITY CAPITAL: EMPIRICAL RESULTS

This section describes the procedures used to estimate the variables for the cross-section regressions (with the exception of β which was dealt with in Section IV), summarizes the regression equations, and presents the cost of equity capital estimates.

(a) Variable Estimation for Cross-Sectional Regression Models 1, 2, and 3

Most of the variables required by estimation models 1, 2, and 3 were estimated using trend projections. The statement that a variable is a trend value means that it is equal to the estimated value for the current period derived from a regression analysis of the variable against time. This smoothing is done to reduce potential errors in measurement. The reasoning is that annual observations of a variable can be viewed, to a first approximation, as observations about a trend consisting of a true component and a random element. If these random elements have zero mean and are serially uncorrelated, use of the trend value of the variable, rather than the current observed value, will tend to reduce errors in measurement. In computing the trend values, all available data up to the estimation date are included (a maximum of 13 observations for 1974 estimates).

The variables to be estimated are: (i) the future growth rate of dividends, g; (ii) the dividend and earnings yields, D_1/P_0 and E_1/P_0; (iii) the rate of return on reinvested earnings, r; and (iv) the investment ratio I_1/P_0.

i. Growth Rate of Dividends (g)

What is required is an estimate of how the market expects dividends per share will grow in the future. However, we have no choice but to try to impute this figure from past growth rates. Since dividends tend to be smoothed by management, it is customary to measure instead the growth of past earnings. This procedure will typically produce a more reliable estimate of how dividends will grow in the future. The model of earning growth typically used is given by:

$$E_t = E_{61}(1 + g)^t \qquad (4\text{-}11)$$

$$t = 1, \ldots, \text{maximum } 13$$

where E_t is the earnings per share in year 1961 + t (1962 is the starting date on the PDE tape). Taking logarithms of both sides results in a regression equation which is linear in t.

$$\log E_t = \gamma_0 + \gamma_1 \cdot t \qquad (4\text{-}12)$$

where $\gamma_0 = \log E_{61}$ and $\gamma_1 = \log (1 + g)$. Equation (4-12) was fitted to the historical earnings data for each company and estimates of the growth rates obtained. In order to be included in the primary sample, at least four positive annual earnings observations were required prior to the estimation date. Years with negative earnings were deleted from the regression analysis.

In addition to the estimate of future growth rates, the above regression provides another important estimate——the standard error of the growth rate. The standard error is a measure of the amount of information in our forecasts of future dividends. A large standard error indicates that very little can be learned from past growth about future growth. Hence, a cost-of-capital estimate based on projection of past trends may be subject to wide error. Conversely, if the standard error of the past growth rate is small, our cost-of-capital estimate will likely be a reasonable proxy for the market's required rate of return. The standard error is consequently used as a weighting factor in the cross-sectional regressions. The application of these weights is discussed in subsection (b) below.

ii. Dividend and Earnings Yields, (D_1/P_0) and (E_1/P_0) Respectively

These values are the trend values of dividends and earnings per share divided by the price as of the estimation date. For example, the 1974 cross-section regressions use the trend value 1975 estimate for D and E divided by the price per share as of December 31, 1974.

iii. The Rate of Return on Reinvested Earnings, r

The only practical alternative for estimating r is through the book rate of return figures. The values used were the trend values of earnings per share divided by the book value per share at the beginning of the year.

iv. Investment Ratio, (I_1/P_0)

The values used were the trend values of the change in book value per share during the year to the stock price at the beginning of the year. The change in book value is essentially the retained earnings during the year.

(b) Estimation of Weights for Cross-Sectional Regression Equations

As described in Section II, a weighted regression procedure was used to estimate the coefficients of Equations (4-2) through (4-4). The weights are the reciprocals of the standard errors of the estimated earnings growth rates. They are estimated along with g from Equation (4-12).

Exhibit 4-1 shows a plot of the December 1974 weights versus stock betas. The weights have been normalized (i.e.,

```
XMEAN=  1.010   XSD=  0.325   YMEAN=  1.000   YSD=  1.227   CORR.=  -0.234   NOBS= 200
INTERCEPT=  1.889   SLOPE=  -0.882         CELL SIZES..  X=  0.0237    Y=  0.2322
```

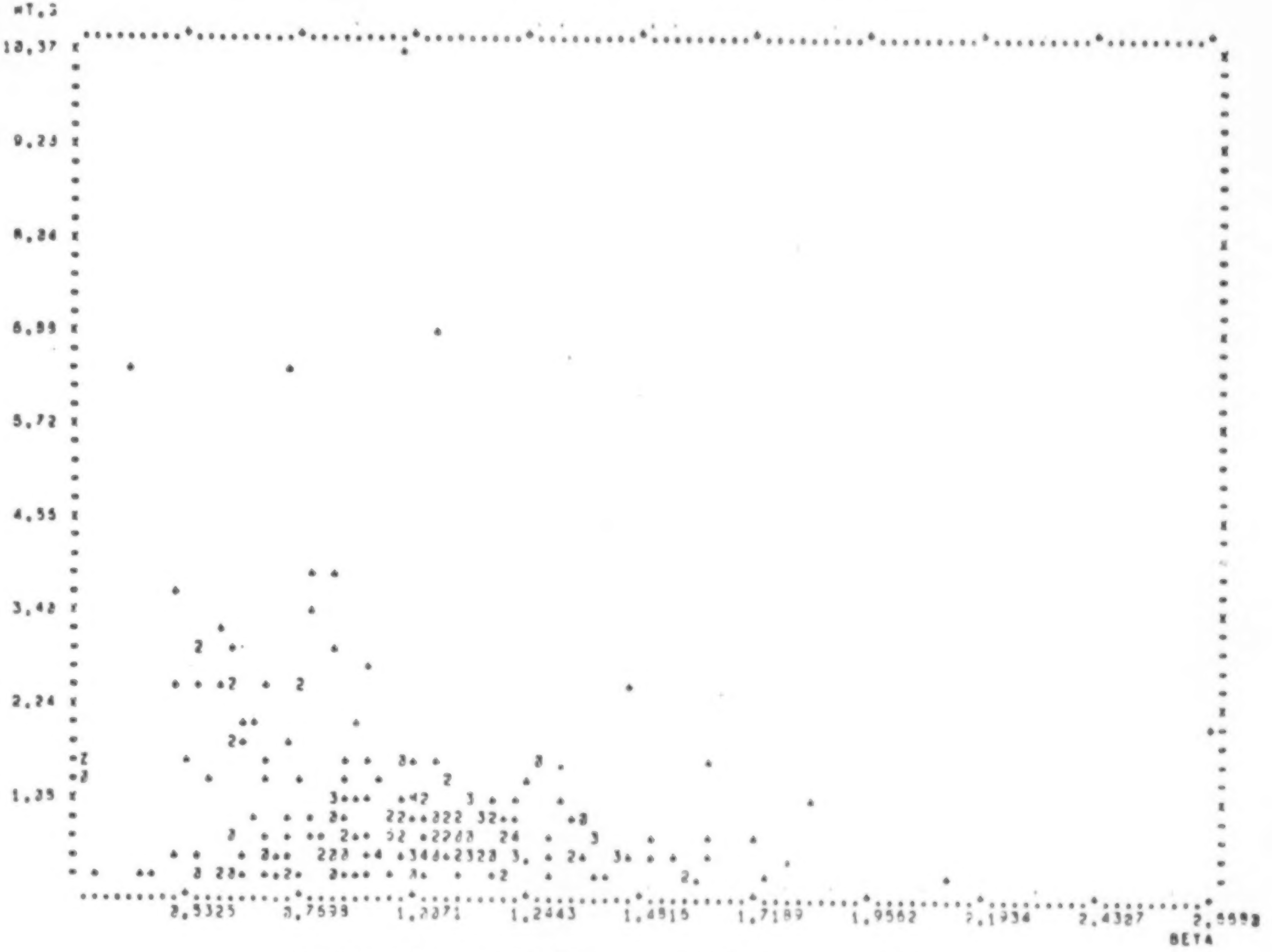


Exhibit 4-1. Normalized regression weights versus beta.

the sum of the weights is equal to the number of companies in the primary sample, 208) to preserve the usual intuitive interpretation of means and regression coefficients.

(c) Weighted Regression Results

The results of the cross-sectional regressions are summarized in Tables 4-4 through 4-8. The tables give the estimated parameter values $(\hat{a}, \hat{b})$, t statistics for the parameter estimates, and R^2 coefficients. The regression results are presented for year-end 1971 through 1974 estimation dates, plus pooled results which combine data from the four periods.

i. Model 1 (see Table 4-4)

The regressions explain roughly 80 percent of the variation in the raw Model 1 cost-of-capital estimates. The intercepts $(\hat{a})$ tend to understate the risk-free rates, and the slopes overstate the likely market risk premiums. (See the last two rows of Table 4-10 for an estimate of the expected market return and the actual risk-free rate.)

ii. Model 2 (see Table 4-5)

The explanatory power of the Model 2 cross-sectional regressions is somewhat less than for Model 1, averaging 66 percent over the four test dates. The intercepts, while some-

TABLE 4-4

WEIGHTED REGRESSION COEFFICIENTS

MODEL 1

$$\left(\frac{D_1}{P_0} + g\right)_j = \hat{a} + \hat{b}\ (\hat{\beta}_j) + \mu_j \qquad j = 1, \ldots, 208$$

YEAR	INTERCEPT		SLOPE		R^2
	$\hat{a}$	t Stat.	$\hat{b}$	t Stat.	
1971	1.27	2.5	11.50	32.3	0.84
1972	1.01	2.0	11.62	32.3	0.84
1973	0.28	0.5	14.0	34.4	0.85
1974	1.17	1.4	16.6	27.8	0.79
POOLED	0.93	2.7	13.42	56.3	0.79

TABLE 4-5

WEIGHTED REGRESSION COEFFICIENTS

MODEL 2

$$\left(\frac{E_1}{P_0}\right)_j = \hat{a} + \hat{b}(\hat{\beta}_j) + \mu_j \qquad j = 1, \ldots, 208$$

YEAR	INTERCEPT		SLOPE		R^2
	$\hat{a}$	t Stat.	$\hat{b}$	t Stat.	
1971	2.78	5.4	6.89	19.2	0.64
1972	2.72	4.9	7.11	18.3	0.62
1973	1.83	2.7	11.27	23.5	0.73
1974	4.11	3.5	15.72	19.2	0.64
POOLED	2.85	6.2	10.25	31.8	0.55

what larger than for Model 1, still tend to understate the government bond rates. The slopes still appear to overstate the market risk premiums, but by less than for the Model 1 results.

iii. Model 3 (see Table 4-6)

The cross-sectional estimation procedures for Model 3 are more complicated, as they involve successive iterations with revised initial estimates of R_j. The results shown in Table 4-6 are from the fourth iteration. After four iterations, the estimated parameters had not significantly changed from the third iteration, and the process was stopped. Two sets of iterations were carried out, one starting with the DCF Model 1 cost-of-capital estimate, Equation (2-8), and the other with the Model 2 estimate, Equation (2-10). The iteration results are summarized in Table 4-7. Table 4-7 shows how the average-cost-of-capital estimate (i.e., the mean $\hat{R}_j$) changed from iteration to iteration. Note that after four iterations the average cost of capital is independent of the starting value.

The intercept ($\hat{a}$) appears to reasonably approximate the risk-free rate. However, the slope ($\hat{b}$) seems low relative to *a priori* expectations about market risk premiums, particularly in 1971 and 1972. The percent of variation explained is approximately 75 percent over the four test dates.

TABLE 4-6

WEIGHTED REGRESSION RESULTS MODEL 3

$$\left(\frac{E_1}{P_0}\right)_j = \hat{a} + \hat{b}(\hat{\beta}_j) + \sum_{k=1}^{8} \hat{C}_k \left[(r - R^*) \frac{I_1}{P_0}\right]_j Z_k + \mu_j \qquad j = 1, \dots, 208$$

$$Z_k = 0 \text{ or } 1$$

YEAR	$\hat{a}$ (t)	$\hat{b}$ (t)	DUMMY VARIABLE COEFFICIENTS (C_k)								R^2
			MINE.	FOR	PAPER	CHEM.	REF.	STEEL	NON-FER	UTIL.	
1971	5.24 (12.2)	2.30 (5.1)	-5.22 (-1.7)	-3.86 (-1.1)	-4.19 (-1.4)	-0.16 (-0.1)	-4.11 (-3.0)	-2.95 (-1.1)	-3.58 (-1.4)	-9.04 (-13.1)	0.82
1972	5.29 (11.2)	2.35 (4.8)	-5.24 (-1.7)	-2.44 (-0.9)	-3.54 (-1.3)	-0.41 (-0.2)	-3.14 (-2.0)	-3.50 (-1.4)	-3.78 (-1.7)	-8.79 (-12.7)	0.80
1973	4.78 (7.5)	6.15 (10.3)	-2.53 (-1.0)	-0.82 (-0.6)	-1.91 (-0.9)	-0.60 (0.0)	-1.50 (-0.9)	-0.25 (-0.1)	-1.31 (-0.5)	-9.30 (-11.6)	0.84
1974	7.72 (5.8)	10.43 (9.7)	-1.82 (-0.8)	-0.25 (-0.2)	-1.16 (-0.6)	-1.01 (-0.5)	0.01 (0.0)	-0.86 (-0.5)	-2.96 (-0.9)	-9.63 (-7.0)	0.71
POOLED	5.24 (11.1)	5.70 (13.0)	-0.55 (-0.4)	0.16 (0.2)	0.01 (0.0)	1.71 (1.4)	0.20 (0.2)	0.34 (0.3)	-0.90 (-0.5)	-8.95 (-14.6)	0.65

TABLE 4-7

MODEL 3 ITERATION SUMMARY

Average Cost of Equity Capital Estimate from Model 3 Cross-Sectional Regression.

STARTING VALUE FOR R_j FOR ITERATION 1	EST. DATE	AVERAGE COST (%/YEAR)			
		ITERATION NUMBER			
		1	2	3	4
$\frac{D_1}{P_0} + g$	1971	9.39	7.86	7.61	7.57
	1972	9.88	8.00	7.72	7.67
	1973	13.78	11.20	11.01	10.99
	1974	18.42	18.08	18.24	18.25
$\frac{E_1}{P_0}$	1971	8.78	7.76	7.60	7.57
	1972	9.07	7.88	7.70	7.67
	1973	13.08	11.13	11.00	10.99
	1974	18.70	18.13	18.24	18.25

iv. Model 4 (see Table 4-8)

The parameters used in Model 4 are summarized in Table 4-8. The model, Equation (4-5), along with the estimates of R_F and R_m were used to produce estimates of the cost of equity capital for each firm in the primary sample for the four test dates plus the pooled interval.

v. Combination Model 1 through Model 4 (see Table 4-9)

Table 4-9 gives the resulting relationship between cost of equity capital and beta when the results of the four cross-sectional models were combined.

Equation (4-8) and Table 4-9 parameter estimates were used to compute the final cost-of-equity capital estimates for the 208 firms in the primary sample. Additionally, Equation (4-8) was used to compute the equity capital costs for firms in the secondary sample that were not included in the primary sample.

(d) The Cost of Equity Capital: Results

The estimated costs of equity capital for the eight primary sample groups are given in Table 4-10. The cost estimates for the 208 firms in the sample were computed from the regression results for Equation (4-8) which are summarized in Table 4-9. The group estimates are unweighted averages of the results for the stocks in the groups. In addition,

TABLE 4-8

PARAMETERS FOR MODEL 4

$$R_j = R_F + \beta_j (R_m - R_F) - (\beta_j - 1)(1 - \delta)(R_m - R_F)$$

δ = Ratio of actual to predicted slope of return-beta line in CAPM tests

= 0.706 (January 31 - June 1970)

YEAR	EXPECTED MARKET RETURN (%/YR) R_m *	EXPECTED RISK-FREE RATE (%/YR) R_F **
1971	13.70	4.35
1972	13.05	5.62
1973	14.14	7.21
1974	17.14	7.01
POOLED	14.51	6.05

* $\left(\frac{D_1}{P_0} + g\right)$ for the Standard and Poor's 500 Stock Composite Index.

** Twelve-month treasury bills, from January.

Source: Solomon Brothers

TABLE 4-9

RELATIONSHIP BETWEEN COST OF EQUITY CAPITAL AND BETA RESULTING FROM COMBINING MODELS 1 THROUGH 4

$R_j = a + b(\hat{\beta}_j)$

a = Imputed risk-free rate

b = Expected risk premium on market $(R_m - R_F)$

YEAR	INTERCEPT $a \equiv (R_F)$	SLOPE $b \equiv (R_m - R_F)$
1971	4.11	6.81
1972	4.22	6.57
1973	4.05	9.07
1974	5.76	12.45
POOLED	4.41	8.83

TABLE 4-10

COST OF EQUITY CAPITAL (% PER YEAR): YEAR END 1971 - 1974

PRIMARY GROUPS

GROUP NAME	SIC CODES	BETA (62-74)	YR END 1971		YR END 1972		YR END 1973		YR END 1974		POOLED (71-74)	
			COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR
MINING (18)	1000-1031	0.94 (0.05)	10.5	1.8	10.4	1.7	12.6	1.2	17.5	1.0	12.7	1.2
FOREST PR. (9)	2400	1.37 (0.08)	13.4	2.8	13.2	2.6	16.5	1.9	22.8	1.9	16.5	1.9
PAPER (31)	2600-2650	0.97 (0.05)	10.7	1.9	10.6	1.7	12.9	1.2	17.8	1.0	13.0	1.2
CHEMICALS (36)	2801-2803	1.22 (0.06)	12.4	2.3	12.2	2.1	15.1	1.5	21.0	1.4	15.2	1.5
REFINING (34)	2911-2913	1.06 (0.06)	11.4	2.0	11.2	1.9	13.7	1.4	19.0	1.2	13.8	1.4
STEEL (28)	3310-3317	1.00 (0.05)	10.9	1.9	10.8	1.7	13.1	1.2	18.2	1.0	13.2	1.2
NON-FER. (17)	3331-3350	1.07 (0.07)	11.4	2.1	11.2	2.0	13.7	1.5	19.1	1.3	13.8	1.5
UTILITIES (35)	4911-4912	0.70 (0.02)	8.9	1.4	8.8	1.3	10.4	1.1	14.5	0.8	10.6	1.0
TOTAL (208)	--	1.01 (0.02)	11.0	1.8	10.9	1.6	13.2	1.2	18.3	1.0	13.3	1.2
EXPECTED MKT. RETURN	--	1.0	10.9	1.8	10.8	1.6	13.1	1.2	18.2	1.0	13.2	1.2
12-MO RISK-FREE RATE	--	0.0	4.35		5.62		7.21		7.01		6.05	

(Standard Error of Mean)

Table 4-10 gives the error ranges associated with the group means. The error ranges for the individual companies and groups were computed using Equation (4-9).

The last two rows of Table 4-10 give the expected market return and risk-free rate as of the end of the test year. The market return, having been produced from Equation (4-8) with beta equal to 1·0, is consistent with the industry costs of capital by construction. The risk-free rates, however, are independent and are measured by the 12-month treasury bill rates. These values are all substantially less than the expected market returns, as would be expected.

The results for the pooled interval are shown graphically in Exhibits 4-2 and 4-3. Exhibit 4-2 shows the estimates for the 208 firms versus their beta coefficients. Since the combined equity estimates were produced from Equation (4-8), the resulting scattergram is a straight line as shown. The error ranges in Exhibit 4-3 show a more complex pattern. the error ranges tend to decrease as beta moves from 0 to 1, and they increase sharply. Firms with the highest betas tend to have the highest standard errors of beta and hence larger error ranges.[6]

```
XMEAN=   1,010    XSD=   0,325    YMEAN=  13,320    YSD=   2,875    CORR.=   1,000    NOBS= 200
INTERCEPT=   4,405    SLOPE=   8,825          CELL SIZES..  X=  0,0237     Y=  0,4648
```

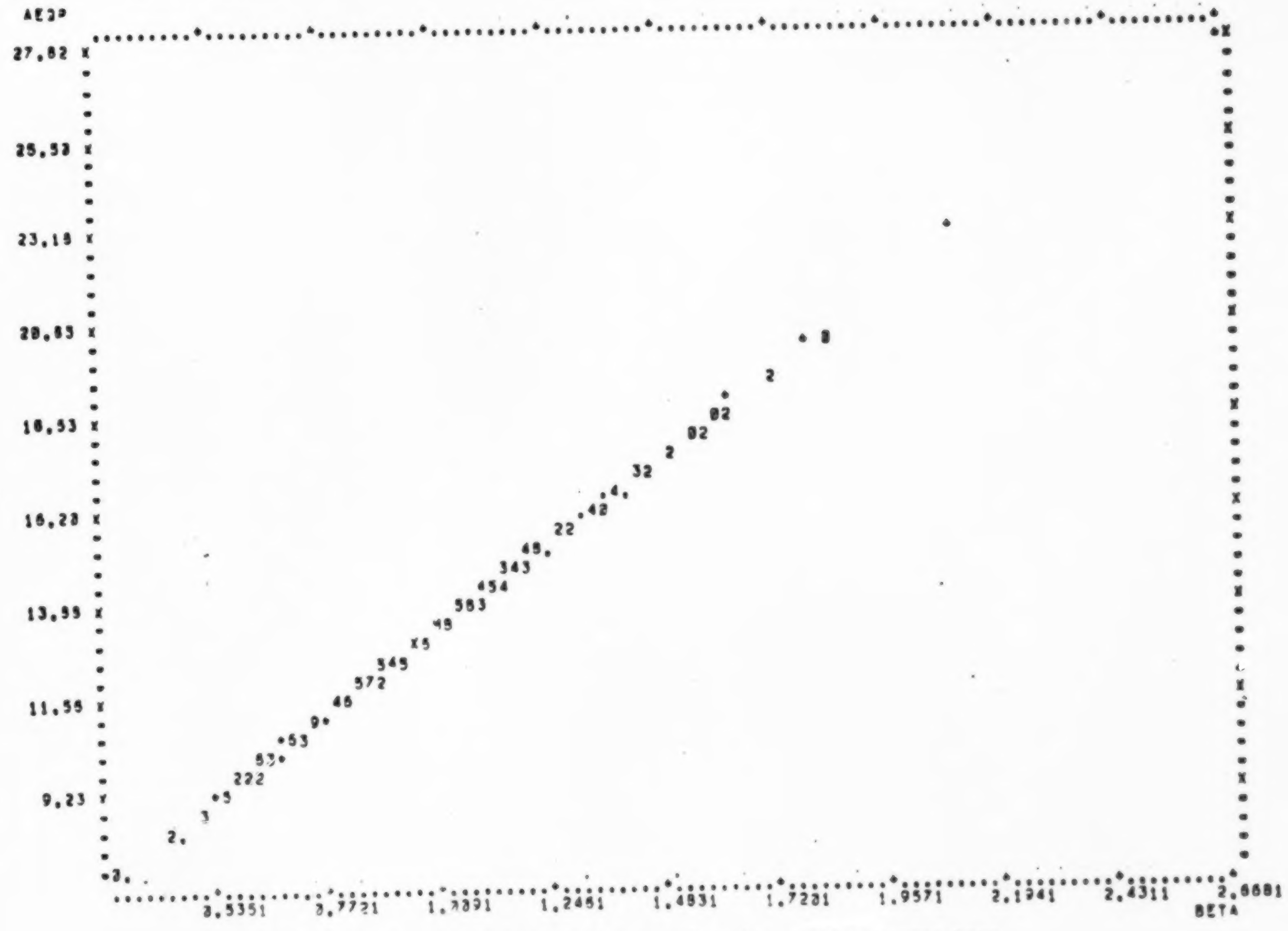


Exhibit 4-2. Pooled estimated cost of equity capital versus beta.

```
XMEAN=  1,010   XSD=  0,325   YMEAN=  2,215   YSD=  0,768   CORR.=  0,727   NOBS= 288
INTERCEPT=  0,483   SLOPE=  1,714        CELL SIZES.. X=  0,0237    Y=  0,1322
```

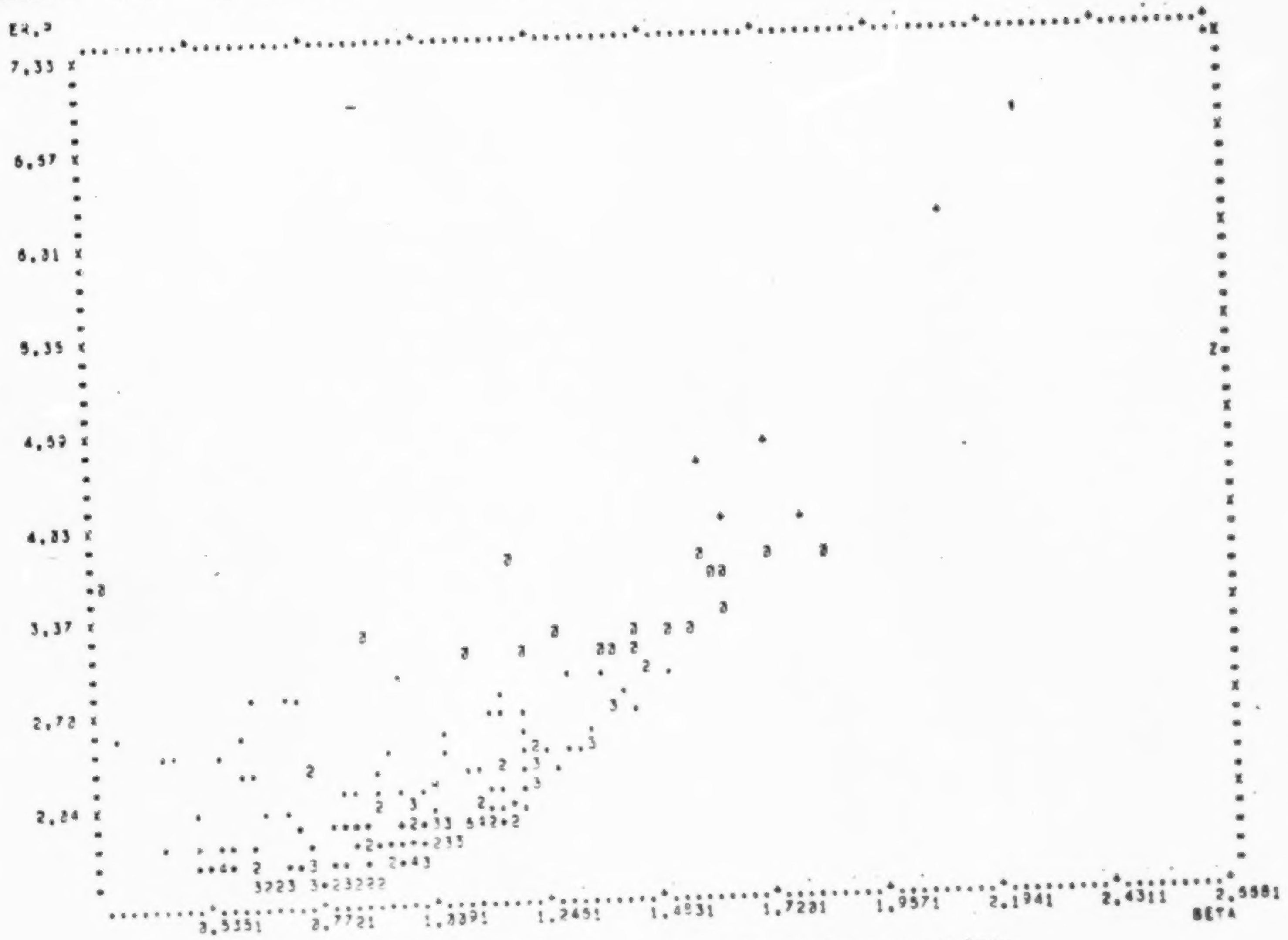


Exhibit 4-3. Error range for pooled equity estimate versus beta.

VI. THE WEIGHTED AVERAGE COST OF CAPITAL: EMPIRICAL RESULTS

Section VI describes the methods used to estimate the cost of debt and preferred stock capital for sample firms, to compute the market value proportions of the firms' capital structures, and to compute the weighted average costs of capital. Empirical results are presented for the 8 primary and 26 secondary groups.

(a) The Cost of Debt and Preferred Stock Capital

The cost of debt capital to a firm is simply the yield to maturity on its outstanding debt. This assumes, of course, that the firm would be able to borrow additional debt capital at this rate.

It is a fairly straightforward matter to compute the yield to maturity on any bond which is traded on an exchange. For example, assume a bond has a market value V_0, principal value P, maturity date T years in the future, and annual interest payments of C dollars per year. Then the yield to maturity, i, is found by solving the equation

$$V_0 = \sum_{t=1}^{T} \frac{C}{(1+i)^t} + \frac{P}{(1+i)^T} \qquad (4\text{-}13)$$

In principal, this calculation could be performed for each bond issue and firm in our sample. But the data collection requirements of this approach make it impractical.

A second approach would be to use the embedded debt cost to the firm. Embedded debt costs (i.e., interest paid divided by the book value of debt) are available in machine-readable form on the Compustat Annual Industrial Tape.[7] But these debt costs will understate the cost of debt capital since most corporate debt has been issued in times of lower interest rates (and hence the market values of debt outstanding are less than the book values).

The approach used in this study is midway between these extremes. An aggregate bond risk rating was prepared for each sample firm as of the years ending 1971 through 1974. The bond ratings were obtained from the January issues of the Standard and Poor's Bond Guides. The S&P bond risk ratings ranged from AAA (the highest quality) to BB (the lowest quality) for bonds of the sample firms. When a firm had two or more bond issues with different S&P ratings (e.g., A and BBB), a weighted rating was constructed based on the relative amount of the bonds outstanding. When a firm's bonds were not rated by Standard and Poors, they were assigned the average rating of firms in their primary group. For example, a paper company with unrated bonds would be assigned the rating of the average of paper companies with ratings available.

Once a bond risk rating was assigned to each sample firm, the cost of debt capital was then given by the yield to maturity on the corresponding Standard and Poor's Bond Index. For example, a utility with an AA risk rating was assigned the current yield on the S&P AA public utility index. When a firm had a composite bond risk index, a composite bond yield was computed using the same proportions.

The advantages of this approach are that it is feasible to produce statiscally unbiased estimates of debt capital costs without an enormous effort. The degree of approximation is substantially lessened as we shift our attention to groups of firms.

For preferred stock, a similar process was contemplated. However, since preferred stock is a relatively insignificant portion of the capital structure of all the industry groups with the exception of utilities, the significant effort required would have borne small return. Further, most of the issues tend to be of high grade, particularly in the case of public utilities. Thus, for preferred stock a simpler approach was used. An index of yields on high-grade corporate and public utility stocks was used to measure the current market costs of preferred stock for all firms.

(b) The Capital Structure Proportions

Computing the proportions of various types of financing in the firm's capital structure would be relatively simple if market values were available for each type of capital. But this is not the case.

The Compustat Annual Industrial Tape, which provides the data base for the capital structure calculations, has market values only for the common equity portion (number of shares outstanding time price per share). The tape contains book values for debt and preferred stock. The problem is to convert these latter figures to market values.

The market values corresponding to the book values were estimated by using the relationship between the embedded financing cost and the current cost, as determined in Section VI-a. A factor was computed for each firm and type of financing which converted the book values to estimated market values. The factor, F, is given by

$$F = \sum_{t=1}^{T} \frac{d}{(1 + i)^{t}} + \frac{1.0}{(1 + i)^{T}} \qquad (4\text{-}14)$$

where d is the embedded debt cost per dollar of book debt, T is average maturity of the firm's debt, and i is the current cost of capital. For preferred stock, T is typically infinite; for bonds we used the average maturity of all outstanding corporate bonds (about 11 years). This procedure is again statistically unbiased and will produce better estimates for groups rather than individual stocks.

Another thorny problem involves the question of leases. Leases provide an alternative form of debt financing. As such, they should be capitalized and considered as an integral part of the debt structure of the firm. This was done in the study by assuming that current lease payments would continue indefinitely, and then capitalizing these payments at the cost of debt capital. This was done for each of the four test years for every sample firm with lease payments.

The capital structure proportions were computed for each firm in the primary and secondary samples for each of the four test years. Additionally, a pooled value was computed for each firm by averaging the four annual values. These pooled values are shown in Table 4-2 for the 8 primary group averages, and in Table 4-3 for the 26 secondary subgroup averages (last four columns).

(c) The Weighted Average Cost of Capital

The groundwork has now been fully laid for the calculation of the weighted average cost of capital, which we have designated as ρ (the j subscripts denoting the firm number have been deleted for ease of exposition). For each firm, the formula for ρ^*, as given in Part 3 is

$$\rho^* = i(1 - T)\frac{D}{V} + i(1 - T)\frac{L}{V} + k_p\frac{P}{V} + R\frac{E}{V} \qquad (2\text{-}27)$$

(See Part 2, Equation (2-27) for definition of symbols.)

The cost of equity capital R was measured in Section V; the remaining variables were estimates, as previously discussed in this section.

Further, we can estimate an error range for the weighted average cost of capital estimates. The major source of error in these estimates is the error of the cost of equity estimates. The errors associated with estimation of the other variables in the ρ^* formula are relatively smaller, and tend to wash out as we shift our attention to groups of stocks. The error range for $\hat{\sigma}$ (which we shall designate as $\hat{\sigma}_T$) is given by

$$\hat{\sigma}_T = \left(\frac{E}{V}\right) \cdot \hat{\sigma} \qquad (4\text{-}15)$$

where $\hat{\sigma}$ is the error range for the cost of equity capital (see Equation (4-9)).

The empirical results are presented in Tables 4-11 and 4-13 Table 4-11 presents the cost estimates and error ranges for the eight primary sample groups. Table 4-13 (first four numerical columns) gives the weighted average cost estimates for the 26 secondary subgroups.

The pooled total cost estimates for the 208 primary sample firms are plotted versus their common stock betas in Exhibit 4-4. The weighted cost estimates tend to

TABLE 4-11

WEIGHTED AVERAGE COST OF CAPITAL (% PER YEAR): YEAR END 1971 - 1974

PRIMARY GROUPS

GROUP NAME	SIC CODE	MKT VALUE DEBT/ TOT. ASSETS	YR END 1971		YR END 1972		YR END 1973		YR END 1974		POOLED ('71-'74)	
			COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR
MINING (18)	1000-1031	13.4	9.3	1.5	9.3	1.4	11.2	1.0	15.4	0.8	11.5	1.0
FOREST PR. (9)	2400	33.0	10.1	1.8	9.8	1.7	11.7	1.2	15.8	1.2	12.0	1.2
PAPER (31)	2600-2650	33.7	8.1	1.2	8.2	1.1	9.3	0.7	12.5	0.6	9.8	0.8
CHEMICALS (36)	2801-2803	37.0	9.1	1.4	9.0	1.3	10.1	0.8	13.5	0.8	10.5	0.9
REFINING (34)	2911-2913	34.0	8.4	1.3	8.5	1.2	10.3	0.9	13.9	0.8	10.2	0.9
STEEL (28)	3310-3317	35.6	8.1	1.1	8.2	1.1	9.6	0.7	12.6	0.6	9.6	0.8
NON-FER. (17)	3331-3350	35.2	8.4	1.3	8.1	1.2	9.4	0.8	12.7	0.8	10.2	0.9
UTILITIES (35)	4911-4912	44.1	6.5	0.7	6.3	0.6	6.8	0.4	8.7	0.3	7.3	0.5
TOTAL (208)	--	34.7	8.3	1.2	8.2	1.1	9.5	0.8	12.7	0.7	9.9	0.8
EXPECTED MKT. RETURN	--	--	10.9	1.8	10.8	1.6	13.1	1.2	18.2	1.0	13.2	1.2
12-MO. RISK-FREE RATE	--	--	4.35		5.62		7.21		7.01		6.05	

```
XMEAN=  1.010   XSD=  0.325   YMEAN=  9.882   YSD=  2.445   CORR.=  0.717   NOBS= 200
INTERCEPT=  4.445   SLOPE=  5.382          CELL SIZES..  X=  0.0237    Y=  0.2853
```

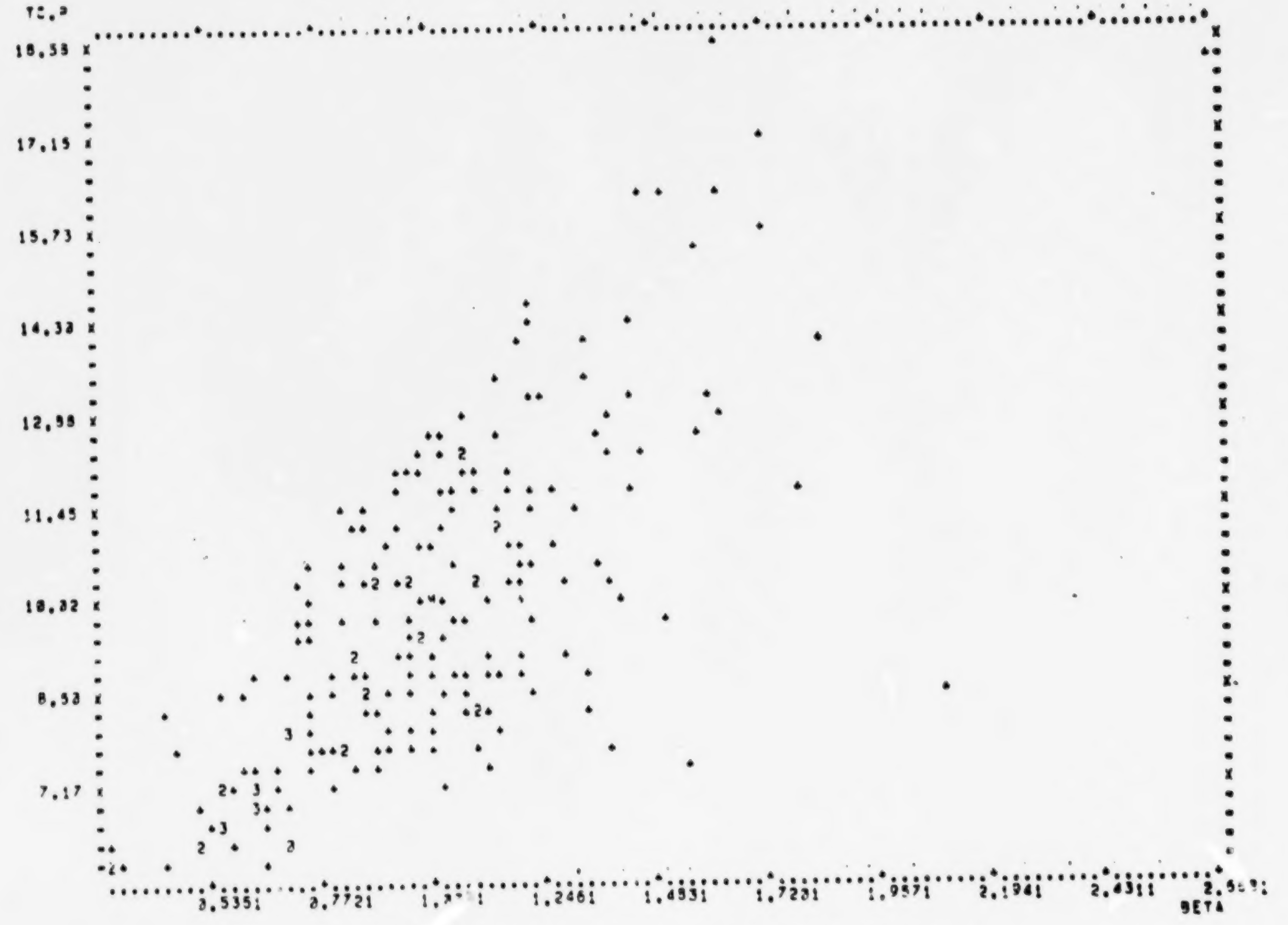
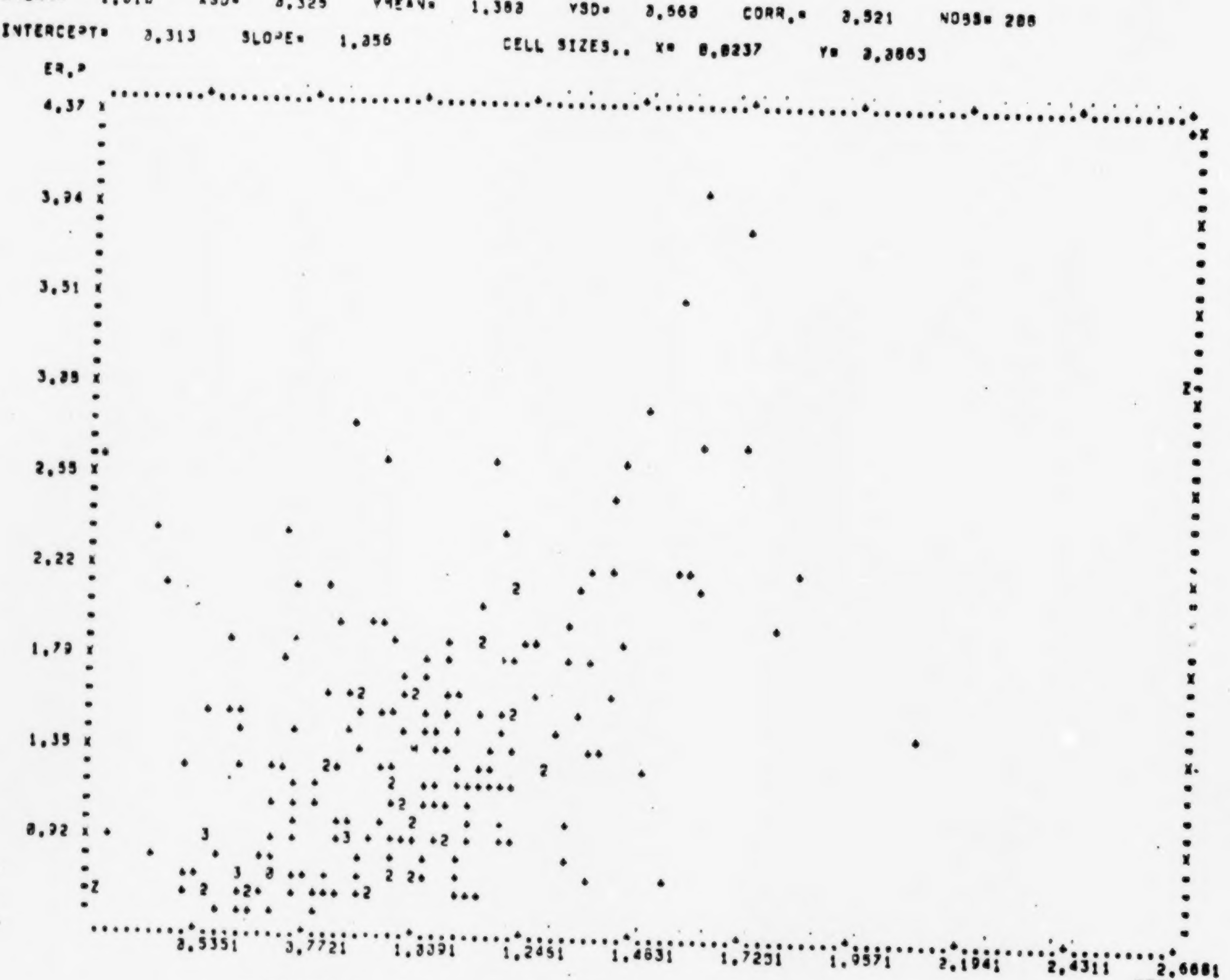


Exhibit 4-4. Pooled estimated total cost of capital versus beta.

increase with stock betas, but the correlation is less than perfect, as would be expected. The error ranges for these cost estimates are shown in Exhibit 4-5.



```
XMEAN=  1,010    XSD=  0,325    YMEAN=  1,380    YSD=  0,560    CORR.=  0,521    NOBS= 200
INTERCEPT=  0,313    SLOPE=  1,056          CELL SIZES..  X=  0,0237     Y=  0,0863
```

Exhibit 4-5. Error range for pooled total cost estimate versus beta.

VII. FORECASTING THE WEIGHTED AVERAGE COST OF CAPITAL

Forecasting the manner in which capital costs are likely to change over time is an extremely hazardous business. Many problems are involved, not the least of which is the need to predict general movements in stock market prices.

The total cost of capital for a firm is usually considered as having three components: the real rate of interest, an inflation premium, and a risk premium. That is,

$$\rho^* = \begin{array}{c}\text{Real Rate}\\ \text{of}\\ \text{Interest}\end{array} + \begin{array}{c}\text{Inflation}\\ \text{Premium}\end{array} + \begin{array}{c}\text{Risk}\\ \text{Premium}\end{array} \qquad (4\text{-}16)$$

The government bond rates, and in particular the shorter term rates, are usually assumed to be estimates of the first two components. Thus, given forecasts of government bond rates, we would have at least part of the problem solved.

This leaves the really hard part——how to predict changes in risk premiums over time. There is no satisfactory way to do this. About all that can be done is to assume that future values will reflect an average of past risk premiums.[8]

Forecasts of future capital costs were prepared using Equation (4-16) and are presented in Tables 4-12 (the 8 primary groups) and 4-13 (the 26 secondary subgroups). These forecasts are conditional on the following assumptions:

1. The interest rate forecasts will reasonably reflect changes in the real rate of interest

and the inflation during the 1975-1984 period for which forecasts are being made. The rate forecasts were for 12-month government bonds and were supplied by Chase Econometrics.

2. The risk premiums required by investors during the forecast period are equal to the average estimated premiums during the 1971-1974 period. These premiums are contained in the pooled cost estimates and are estimated by subtracting the 12-month interest rates. This assumption may not seem so unrealistic when market conditions during the 1971-1974 period are considered. In the early years, market levels were high and risk premiums relatively low. At the end of 1974, market levels were low and observed risk premiums appeared to be at an all-time high. Thus, the pooled risk premium estimates will reflect somewhat average conditions and provide a not-unrealistic basis for the forecasts.

The reader should keep in mind that the forecasts given in Tables 4-12 and 4-13 are not for 1-year periods but long-run estimates as of the estimation date. For example, the 12.4% forecast made for Forest Products as of December 31, 1975 (see Table 4-12, column 1, row 2) is an estimate of the long-run cost of capital. It is not possible, given the current state of finance theory, to say how this aggregate rate can be broken down into a series of annual discount rates (i.e., a rate for 1975, for 1976, etc.). This would require a term structure theory for stock prices which, to date, has not appeared.

TABLE 4-12

PROJECTED WEIGHTED AVERAGE COST OF CAPITAL (% PER YEAR): YEAR END 1975 - 1984

PRIMARY GROUPS

GROUP NAME	1975		1976		1977		1978		1979		1980		1981		1982		1983		1984	
	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR	COST	ERROR
MINING (18)	11.8	1.5	13.1	1.8	13.2	2.1	14.5	2.3	14.8	2.5	13.7	2.7	12.5	2.9	11.9	3.1	12.1	3.2	12.5	3.4
FOREST PR. (9)	12.4	1.8	13.7	2.2	13.8	2.5	15.1	2.8	15.4	3.0	14.3	3.3	13.1	3.5	12.5	3.7	12.6	3.9	13.0	4.1
PAPER (31)	10.1	1.1	11.4	1.4	11.5	1.6	12.8	1.7	13.1	1.9	12.0	2.1	10.8	2.2	10.2	2.3	10.4	2.5	10.8	2.6
CHEMICALS (36)	11.1	1.3	12.4	1.6	12.5	1.8	13.8	2.0	14.1	2.2	13.0	2.4	11.8	2.6	11.2	2.7	11.4	2.9	11.8	3.0
REFINING (34)	10.5	1.2	11.8	1.5	11.9	1.8	13.2	2.0	13.8	2.2	12.4	2.3	11.2	2.5	10.6	2.6	10.8	2.8	11.2	2.9
STEEL (28)	10.2	1.1	11.5	1.3	11.6	1.5	12.9	1.7	13.2	1.9	12.1	2.0	10.9	2.1	10.3	2.3	10.4	2.4	10.8	2.5
NON-FER (17)	10.5	1.3	11.8	1.6	11.9	1.9	13.2	2.1	13.5	2.3	12.4	2.5	11.2	2.7	10.6	2.8	10.8	3.0	11.2	3.1
UTILITIES (35)	7.7	0.7	9.0	0.8	9.1	0.9	10.4	1.1	10.7	1.2	9.6	1.3	8.4	1.3	7.8	1.4	7.9	1.5	8.3	1.6
TOTAL (208)	10.2	1.2	11.5	1.4	11.6	1.6	12.9	1.8	13.2	2.0	12.1	2.2	10.9	2.3	10.3	2.5	10.5	2.6	10.9	2.7
EXPECTED MKT. RETURN	13.6		14.9		15.0		16.3		16.6		15.5		14.3		13.7		13.8		14.2	
EXPECTED RISK-FREE RATE	6.40		7.71		7.80		9.10		9.40		8.30		7.10		6.50		6.65		7.05	

TABLE 4-13

WEIGHTED AVERAGE COST OF CAPITAL (% PER YEAR)

SECONDARY GROUPS

GROUP		SUBGROUP	ESTIMATED					PROJECTED									
			1971	1972	1973	1974	POOLED	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
PULP AND PAPER	1	LARGE MULTIPRODUCT COMPANIES (8)	8.3	8.4	10.0	13.5	10.1	10.4	11.8	11.8	13.1	13.4	12.3	11.1	10.5	10.7	11.1
	2	MEDIUM WITH RESOURCE CONTROL (19)	7.4	7.5	8.6	11.6	9.0	9.3	10.6	10.7	12.0	12.3	11.2	10.0	9.4	9.6	10.0
	3	SMALL-MEDIUM W/O RESOURCE CONTROL (10)	7.2	7.3	8.0	10.8	8.7	9.0	10.3	10.4	11.7	12.0	10.9	9.7	9.1	9.3	9.7
	4	FOREST PRODUCTS COMPANIES (5)	9.7	9.3	11.4	15.4	11.7	12.0	13.3	13.4	14.7	15.0	13.9	12.7	12.1	12.3	12.7
	5	CONVERTERS (12)	8.5	8.7	8.7	11.5	10.0	10.4	11.7	11.8	13.1	13.4	12.3	11.1	10.5	10.6	11.0
CHEMICALS	6	CHEMICALS—MAJOR (11)	9.1	8.9	10.7	14.5	10.7	11.1	12.4	12.5	13.8	14.1	13.0	11.8	11.2	11.3	11.7
	7	CHEMICALS—INTERMED. (6)	8.3	8.5	9.4	12.4	10.2	10.5	11.8	11.9	13.2	13.5	12.4	11.2	10.6	10.7	11.1
	8	CHEMICALS—SPECIALTY (19)	9.3	9.2	10.0	13.3	11.0	11.4	12.7	12.8	14.1	14.4	13.3	12.1	11.5	11.6	12.0
PETROLEUM REFINING	9	INTEG. MAJORS (14) (200 + B/D)	7.4	7.4	9.2	12.4	8.9	9.3	10.6	10.7	12.0	12.3	11.2	10.0	9.4	9.5	9.9
	10	INTEG. LARGE (5) (200 + B/D)	7.5	7.6	9.1	12.3	8.9	9.3	10.6	10.7	12.0	12.3	11.2	10.0	9.4	9.5	9.9
	11	INTEG. (3) (100 - 200 B/D)	8.6	9.5	11.4	15.5	10.8	11.1	12.4	12.5	13.8	14.1	13.0	11.8	11.2	11.3	11.7
	12	REFINERS (1) (100 - 200 B/D)	7.7	6.3	8.2	10.7	9.0	9.3	10.6	10.7	12.0	12.3	11.2	10.0	9.4	9.5	9.9
	13	INTEG. (2) (70 - 100 B/D)	7.2	7.0	8.6	11.1	8.3	8.7	10.0	10.1	11.4	11.7	10.6	9.4	8.8	8.9	9.3
	14	INTEG. (2) (30 - 70 B/D)	9.0	9.2	10.7	14.3	11.2	11.6	12.9	13.0	14.3	14.6	13.5	12.3	11.7	11.8	12.2
		EXPECTED MKT RETURN	10.9	10.8	13.1	18.2	13.2	13.6	14.9	15.0	16.3	16.6	15.5	14.3	13.7	13.8	14.2
		12-MO RISK-FREE RATE	4.4	5.6	7.2	7.0	6.1	6.4	7.7	7.8	9.1	9.4	8.3	7.1	6.5	6.7	7.1

TABLE 4-13 (CONTINUED)

WEIGHTED AVERAGE COST OF CAPITAL (% PER YEAR)

SECONDARY GROUPS

GROUP	SUBGROUP	ESTIMATED					PROJECTED									
		1971	1972	1973	1974	POOLED	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
PETROLEUM REFINING (CONT.)	15 INTEG. CAN. (2) (30 - 70 B/D)	6.3	6.6	7.8	10.3	7.5	7.7	9.0	9.1	10.4	10.7	9.6	8.4	7.8	7.9	8.3
	16 REFINERS (2) (30 - 70 B/D)	8.8	8.1	10.3	13.9	11.0	11.4	12.7	12.8	14.1	14.4	13.3	12.1	11.5	11.6	12.0
	17 REFINERS (3) (10 - 30 B/D)	11.8	10.4	12.7	17.1	14.4	14.7	16.0	16.1	17.4	17.7	16.6	15.4	14.8	15.9	16.3
	18 INTEG. (1) (10 - 30 B/D)	13.6	13.2	16.5	22.5	16.3	16.7	18.0	18.1	19.4	19.7	18.6	17.4	16.8	16.9	17.3
IRON AND STEEL	19 STEEL—MAJOR (7)	7.3	7.5	8.8	11.8	9.0	9.3	10.6	10.7	12.0	12.3	11.2	10.0	9.4	9.6	10.0
	20 STEEL—MINOR (20)	8.3	8.4	9.8	13.1	10.1	10.4	11.7	11.8	13.1	13.4	12.3	11.1	10.5	10.7	11.1
NON-FERROUS METALS	21 PRIMARY COPPER (8)	9.6	9.6	11.8	16.1	12.0	12.4	13.7	13.8	15.1	15.4	14.3	13.1	12.5	12.6	13.0
	22 PRIMARY LEAD AND ZINC (3)	8.7	8.5	10.2	13.7	10.5	10.9	12.2	12.3	13.6	13.9	12.8	11.6	11.0	11.1	11.5
	23 PRIMARY ALUMINIUM (4)	6.5	6.4	8.1	10.6	8.2	8.6	9.9	10.0	11.3	11.6	10.5	9.3	8.7	8.8	9.2
	24 SECONDARY SMELTING (4)	7.1	6.9	8.5	11.3	9.0	9.3	10.7	10.7	12.0	12.3	11.2	10.0	9.4	9.6	10.0
UTILITIES	25 UTILITIES FLOW THRU (19)	6.3	6.1	6.7	8.4	7.1	7.5	8.8	8.9	10.2	10.5	9.4	8.2	7.6	7.7	8.1
	26 UTILITIES NORMALIZED (16)	6.7	6.5	7.0	9.0	7.6	8.0	9.3	9.4	10.7	11.0	9.9	8.7	8.1	8.2	8.6
	EXPECTED MKT RETURN	10.9	10.8	13.1	18.2	13.2	13.6	14.9	15.0	16.3	16.6	15.5	14.3	13.7	13.8	14.2
	12-MO RISK-FREE RATE	4.4	5.6	7.2	7.0	6.1	6.4	7.7	7.8	9.1	9.4	8.3	7.1	6.5	6.7	7.1

FOOTNOTES FOR PART 4

1. The numerical results presented in Part 4 are taken from an unpublished working paper by Professor Gerald A. Pogue entitled "The implications of Modern Finance Theory for Estimating the Cost of Capital" [25], dated August 1975.

2. There is no need to assume that the variances of ϵ_j are the same for all firms. Indeed, the $\hat{R}_j$ estimates from the discounted cash flow models will convey different amounts of information about the true values, R_j. This results in a heteroscedasticity problem which is handled by using weighted regression analysis (see Section II(d)).

3. This averaging procedure will reduce the measurement error for $\hat{R}_j$ as long as the error variances of the four individual estimates are roughly comparable in size and not perfectly correlated. The optimal weighting scheme requires knowledge of the (unobservable) error covariable matrix for the four estimates ($\hat{R}_j^{\ell}$, $\ell = 1, 2, 3, 4$). With this knowledge an optimal set of weights can be derived for each company to minimize the error of the combined estimate R_j.

4. An alternative weighting procedure to (1/4, 1/4, 1/4, 1/4) would recognize the complementary nature of estimates 1 and 2 and give equal weight to estimates 3 and 4 and a combination of 1 and 2 (e.g., 1/6, 1/6, 1/3, 1/3).

5. The PDE tape is produced by Investors Management Sciences, Inc. (a subsidiary of Standard and Poor's Corporation). The January 1975 version of the tape was used. The tape contains monthly data for approximately 3,000 corporations for the 1962-1974 period. The data items include stock prices, dividends, earnings, and common stock book values.

6. This effect is largely due to construction. The data means for the four estimation equations tend to cluster near $\beta = 1.0$. Thus the dispersion of the four $\hat{R}_j$ estimates tends to increase as $|\beta_j - 1|$ increases.

7. The Compustat Annual Primary Industrial File contains balance sheet and income statement data for approximately 3,000 industrial and utility firms. It is produced by Investors Management Sciences, Inc., a subsidiary of the Standard and Poor's Corporation.

8. An average of the four risk premiums is a more efficient estimator of future risk premiums than the last (1974)

observation. The four observations can be viewed as independent draws from the underlying risk premium population. Thus, the best predictor of the next value is an average rather than the last value.

ESTIMATION OF THE COST OF CAPITAL FOR MAJOR UNITED STATES INDUSTRIES WITH APPLICATION TO POLLUTION-CONTROL INVESTMENTS

PART 5. APPLICATIONS

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November 1975

I. INTRODUCTION

The purpose of Part 5 is to show how the cost-of-capital estimates developed in Part 4 can be used for making capital budgeting decisions in practical situations. Section II reviews and extenda the capital budgeting procedures introduced in Part 2. Section III develops the basic structure for an analytical framework which can be used for measuring the financial impact on corporations of investments in pollution-control devices.

II. CAPITAL BUDGETING

(a) The Weighted Average Cost of Capital

In finance theory it is usually assumed that the goal for financial decision-making is to increase the wealth of corporate stockholders. This results in the following decision rule for evaluating corporate investment alternatives: Accept only projects which have positive net present values. The net present value (NPV) is the discounted sum of net project cash flows (inflows minus outflows). The discount rate is the weighted average cost of capital.

$$NPV(\rho^*) = \sum_{t=0}^{T} \frac{C_t}{(1+\rho^*)^t} \qquad (5\text{-}1)$$

where

C_t = the expected net cash flow in year t

ρ^* = the weighted average cost of capital

T = the economic lifetime of the project

The decision rule for corporate managers is

$$\text{Accept Project if } NPV(\rho^*) > 0 \qquad (5\text{-}2)$$

The principal unknown in Equation (5-1) is the weighted average cost of capital, ρ^*. As discussed in Part 2, ρ^* under certain circumstances can be estimated using a rule known as the textbook formula.

$$\rho^* = i(1 - T_c) \cdot \frac{D}{V} + R \cdot \frac{E}{V} \qquad (5\text{-}3)$$

where

i = the current interest rate on the firm's bonds

R = the cost of equity capital

D = the market value of debt (bonds plus leases)

E = the market value of equity

T_c = the corporate tax rate

V = $D + E$

Equation (5-3) provides a reasonable estimate of the weighted average cost of capital only under certain conditions. The major requirements are (1) the project does not change the business risk of the firm—it must be an average-risk investment, (2) the project does not shift the firm's debt ratio—it must be financed with the same proportion of debt and equity as the overall firm, and (3) the project must make a permanent contribution to debt capacity—the project must be a perpetuity and support perpetual debt. (See Part 2, Section IV-b for discussion of these assumptions.)

The Adjusted-Present-Value Approach

An alternative to Equations (5-1) and (5-2) is the adjusted-present-value (APV) approach.[1] The adjusted present value is defined by

$$APV = \sum_{t=0}^{T} \frac{C_t}{(1+\rho_0)^t} + PVTS \qquad (5\text{-}4)$$

where

C_t = the expected net project cash flow in year t (same as in Equation (5-1))

T = economic lifetime of the project (years)

PVTS = the present value of the tax shield on debt supported by the project

ρ_0 = the all-equity cost of capital—the cost of capital if the project were all-equity financed

The capital budgeting decision rule is

$$\text{Accept Project if APV} > 0 \qquad (5\text{-}5)$$

The APV rule accomplishes in two parts what the NPV approach attempts in one. The APV is the sum of the project's net present value under the assumption of all-equity financing, plus the present value of the tax shield on debt. Since both components are dealt with separately, no restrictive assumptions on the amount and duration of the project and debt cash flows are required. The NPV rule, on the other hand, attempts to deal with both components at once and results in a complicated definition of ρ^*. Only under restrictive assumptions can ρ^* be simply estimated. For example, ρ^* can be estimated using the textbook rule if the assumptions described in Section II-a are valid.

While the APV is a somewhat more complicated calculation than the NPV, it can result in improved decision-making. Compared with the textbook rule for estimating ρ^*—and hence NPV(ρ^*)——the APV has three distinct advantages. It allows treatment of projects with different business risks, that is, the all-equity rate ρ_0 can be tailored to the project risk. Second, there is no need for the projects to be perpetuities. Third, there is no requirement that the project be financed in a predetermined way, nor that the debt be permanent.

One aspect of the formula left unresolved is the estimation of the all-equity rate. We shall deal with this matter in subsection c.

Example

Long Island Duck Farms (LIDF) has the opportunity to invest \$100,000 ($t = 0$) and expects after-tax cash returns of \$60,000 at $t = 1$ and \$70,000 at $t = 2$. The project will last for two years only. The cost of capital assuming all-equity financing is 12% (ρ_0). The borrowing rate (i) is 8%, and the firm's debt ratio for a project of this type is 0.30. (The target debt ratio is in book value terms.) The problem is to determine if this project has a positive adjusted present value.

We start by computing the project's net present value under the base case assumptions (i.e., all-equity financing).

$$\mathrm{NPV}(\rho_0) = \sum_{t=0}^{2} \frac{C_t}{(1+\rho_0)^t}$$

$$= -100{,}000 + \frac{60{,}000}{1.12} + \frac{70{,}000}{(1.12)^2}$$

$$= \$9{,}375$$

Next we determine the value of the tax shield associated with the debt (PVTS). The book value of the project at $t = 0$ is \$100,000. Thus, \$30,000 extra debt can be issued at $t = 0$, and \$15,000 repaid at $t = 1$ (assuming straight-line depreciation of the asset). Thus,

$$\mathrm{PVTS} = \frac{T_c i D_0}{(1+i)} + \frac{T_c i D_1}{(1+i)^2}$$

$$= \frac{0.5(0.08)(30{,}000)}{1.08} + \frac{0.5(0.08)(15{,}000)}{(1.08)^2}$$

$$= 1{,}111 + 514$$

$$= \$1{,}625$$

Thus, the APV of the project is \$9,375 + \$1,625 = \$11,000.

Suppose we wished to estimate the weighted average cost of capital for this project such that the net present value as computed by Equation (5-1) would also equal \$11,000. The direct way is to solve for ρ^* from Equation (5-1).

$$11{,}000 = -100{,}000 + \frac{\$60{,}000}{(1+\rho^*)} + \frac{70{,}000}{(1+\rho^*)^2}$$

Solving,

$$\rho^* = 11.6\%$$

Applying the textbook formula in a straightforward (but inappropriate) fashion yields $\rho^* = 10.2\%$. If this estimate were used along with Equation (5-1), the resulting NPV would obviously be higher than 11,000. This results because the textbook rule assumes perpetual debt, and thus overstates the present value of the two years of tax shields. Further, the textbook formula assumes the 0.30 debt ratio is in market terms, not book terms, and further overstates the value of debt.

(c) The All-Equity Cost of Capital

At this point it is reasonable to wonder why the textbook formula for computing ρ^* is used at all. One answer is that it provides a helpful starting point for estimating the all-equity rate ρ_0.

The textbook formula can be used to estimate ρ^* for the firm as a whole. Since the firm is assumed to have an indefinite life, and since debt ratios remain reasonably constant, it fits the textbook assumptions better than individual projects.

This rate can then be substituted into the MM cost-of-capital formula to obtain an estimate of ρ_0 for the firm as a whole. The process proceeds as follows:

Step 1—Estimate the textbook ρ^* for the overall firm,

$$\rho^* = i(1 - T_c)\frac{D}{V} + R\frac{E}{V}$$

Step 2—Substitute ρ^* into the MM formula to obtain ρ_0,

$$\rho^* = \rho_0\left(1 - T_c\frac{D}{V}\right) \qquad (5\text{-}6)$$

$$\therefore \rho_0 = \frac{\rho^*}{\left(1 - T_c\frac{D}{V}\right)}$$

(This ρ_0 is applicable only to average-risk projects.)

Step 3—For projects satisfying the MM formula assumptions, use this rule to estimate their ρ_j^* (j denotes project).

$$\rho_j^* = \rho_0(1 - T_c L_j) \qquad (5\text{-}7)$$

where

L_j = the debt-financing ratio for project j

This ρ_j^* can now be substituted into Equation (5-1) to estimate the NPV(ρ^*) for project j.

For projects not satisfying the MM cost-of-capital rule, use the APV approach.

$$APV_j = \sum_{t=1}^{T_j} \frac{C_j t}{(1+\rho_0)} + PVTS_j \qquad (5\text{-}8)$$

Note that we have been careful only to consider average-risk projects for which the ρ_0 is applicable. A thornier problem arises when a project is of different riskiness. It is then necessary to estimate ρ_{0j}—the all-equity rate applicable to project j. There is no simple practical answer to this problem.

When a market risk measure (beta) is available for the project, then the ρ_{0j} can be estimated using the procedures and results of Part 4. That is, given the beta of the project, an estimate of its cost of capital can be estimated. Beta measures, however, are typically not available for individual projects. About the best that can be done is to use the (unlevered) beta for the common stock of a company whose business risk resembles that of the project. For example, a chemical firm contemplating investment in the paper industry could use the average beta for a suitably chosen group of paper companies as an estimate of the market risk of the project. Given the risk estimate, the ρ_{0j} follows as described in Part 4.

If this procedure is used, caution must be taken to ensure that the betas used are unlevered. A company's beta reflects not only its business risk but also its financial risk. To estimate ρ_{0j} we require the all-equity beta (β_0) which reflects only business risk.

The all-equity beta can be estimated from the firm's market beta (β) and the market debt-to-equity ratio $\left(\frac{D}{E}\right)$ as follows.[2]

$$\beta = \beta_0 \left\{ 1 + (1 - T_c) \frac{D}{E} \right\} \qquad (5\text{-}9)$$

$$\therefore \beta_0 = \frac{\beta}{\left\{ 1 + (1 - T_c) \frac{D}{E} \right\}}$$

When market risk measures are not available, a heuristic solution is to position the risk of new projects relative to the average risk. The ρ_{0j}'s for individual projects are then subjectively increased or decreased from the average value. For example, a firm might categorize its investments into three categories—high, average, and low risk—and subjectively adjust the ρ_0 for the high and low groups. While this approach is not very satisfying, it is the one typically used in practice.

Example

Suppose a firm has a market debt ratio of 0.20 and can borrow at 8 percent. The management estimates investors' required rate of return, R, at 12 percent. Then by the textbook formula

$$\rho^* = i(1 - T_c) \frac{D}{V} + R \frac{E}{V}$$

$$= 0.08\ (0.50)(0.20) + (0.12)\ (0.80)$$

$$= 0.104$$

This 10.4-percent rate would be the correct hurdle rate for average-risk perpetual projects with debt financing ratios of 0.20.

The MM formula can be used to estimate ρ_0 for the average-risk project

$$\rho^* = \rho_0\left(1 - T_c \frac{D}{V}\right)$$

$$\therefore \rho_0 = \frac{0.104}{[1 - 0.5(0.20)]}$$

$$= 0.116$$

This is a starting point for using the MM formulas or the APV approach for average-risk projects.

III. MEASURING THE FINANCIAL IMPACT OF INVESTMENTS IN POLLUTION-CONTROL DEVICES

Pollution-control requirements involve long-run sequences of capital investments and annual operating expenses. Consequently, there is a need to measure the magnitude of these requirements relative to the firm's ability to carry them out.

Given the nature of pollution-control investments, their imposition will tend to reduce the value of a firm's assets. How significant is this reduction in value? How much would the return on other corporate assets have to be increased to offset this decline?

The purpose of this section is to introduce a framework for analyzing these questions. This framework will provide a first step toward analyzing the financial impact on corporations of required pollution-control investments. It is not the intention to provide a detailed analysis of such questions, but to point the direction in which such an analysis might proceed. Indeed, many significant questions are left unasked as well as unanswered.

(a) A Financial Impact Index

Pollution-control programs require long-run expenditures by corporations. These include periodic investments in facilities plus annual operating costs. As with other investments, the current economic value of a pollution-control investment (PCI) is the present value of its future cash flows (i.e., the APV). For PCI's, the present values are negative (since all cash flows are negative) and hence they will be undertaken by corporations only if required by law.

A measure of the relative financial impact of a PCI is obtained by comparing its present value to the present value of the firm's other assets. The present value of the firm's assets is given by the market value of the firm (V_B), that is, the market value of the firm's equity and debt obligations prior to adoption of the PCI ($V_B = D_B + E_B$). The relative impact of the PCI can be measured by the ratio of the APV to the value of the firm's other assets. The ratio is designed as the financial impact index (FII).

$$FII = \frac{|APV|}{V_B} \qquad (5\text{-}10)$$

where

$|APV|$ = the absolute value of the (negative) PCI-adjusted present value

Note that the value of the firm after adoption of the PCI (V_A) will equal the prior value V_B plus the APV of the investment.

If the FII is small (e.g., 0.02), then the impact of the PCI on the corporation would similarly be small. However, as the FII increases, the ability of the firm to successfully absorb the PCI will diminish until finally bankruptcy occurs.

Exactly when the critical value of the FII occurs is a difficult question requiring further research.

(b) The Risk and Financing of Pollution-Control Investments

The riskiness and method of financing of pollution-control investments will depend on case-by-case analysis. However, it is possible to define a set of boundary conditions which will delineate the possibilities that will likely exist in practice.

The Risk of PCI's

The maximum risk of a PCI is probably the risk of the firm's other assets. While it is possible the PCI risk would be higher, it is unlikely. It is more likely that the PCI cash flows will not be more variable or less predictable than the firm's other operating cash flows. To the extent that pollution-control costs tend to vary with the firm's business activities, the PCI's will be of average risk.

On the other hand, the minimum possible riskiness will resemble the risk of the firm's debt obligations. Even if the pollution-control costs were contractual, the firm can default by going out of business. Hence, the minimum risk can be no less than that of the firm's debt service obligations.

The Financing of PCI's

The typical PCI will have predominantly negative cash flows (e.g., investment outlays, operating expenses, etc.).

While there will be some favorable side effects (e.g., tax savings from investment depreciation, sale value of biproducts such as sulphuric acid, etc.), these will typically be insufficient to produce either a positive net present value for the PCI or even a significant number of positive annual cash flows.

Given the above, the PCI's will have no inherent debt capacity of their own. In fact, a PCI will reduce the debt capacity of the rest of the firm. This follows because the negative net present values of the PCI's will reduce the ability of the overall firm to support debt service requirements. Thus, the PCI's have negative debt capacity.

For example, a firm which has debt outstanding with a loan covenant requiring maintenance of a certain debt coverage ratio may well have to reduce its debt level given the reduction in operating earnings caused by the PCI.

If a PCI is partially or entirely financed by debt issues, this is possible only because the firm has enough unused debt capacity associated with its other assets (even after the reduction in aggregate debt capacity caused by the PCI) to support the new debt. We must be careful, however, not to attribute the tax benefits of this new debt to the PCI. The firm could have obtained these benefits by using the debt capacity to finance its other projects.

Consider now two limiting cases relating to the firm's overall capital structure, all-equity financed and, the situation where the firm has fully utilized the debt capacity of its existing assets, maintaining a debt ratio equal to (D/V).

In the first case, the reduction in debt capacity caused by adoption of the PCI will have no effect. Since these firms make no use of debt, the present value of tax savings on the debt capacity reduction caused by the PCI (PVTS) has a zero value.

In the second case, the reduction in debt capacity will result in a reduction in the value of the firm. Assuming the firm has fully utilized the debt capacity of its other assets, the adoption of the PCI will require a reduction of the firm's debt level. This follows because, in order to maintain a target debt ratio of, say, 30%, a reduction in the value of the firm's assets due to the PCI will necessitate a reduction in debt level in order to maintain the target debt ratio. The present value of the tax savings on the debt displaced by the PCI is positive or, viewed from the point of view of the PCI, the PVTS of the PCI is negative.

The matrix of possible risk and capital structures is illustrated in Table 5-1. Cases 1 and 2 assume average-risk investments, Cases 3 and 4 assume "low risk". To compute the

TABLE 5-1

POSSIBLE RISK AND CAPITAL STRUCTURE CONSIDERATIONS FOR POLLUTION-CONTROL INVESTMENTS

Investment Risk	Capital Structure of Firm	
	All Equity	Equity & Debt
Average Risk (Same as Other Assets)	Case 1 $\rho_0^{PCI} = \rho_0$ PVTS = 0	Case 2 $\rho_0^{PCI} = \rho_0$ PVTS < 0
Low Risk (Same as Firm's Bonds)	Case 3 $\rho_0^{PCI} = i$ PVTS = 0	Case 4 $\rho_0^{PCI} = i$ PVTS < 0

ρ_0^{PCI} = all-equity rate for pollution-control investments

ρ_0 = all-equity rate of firm's other assets

i = interest rate on firm's bonds

PVTS = the present value of the tax shield on the PCI debt capacity

APV's for these cases, we require estimates of the all-equity financing rate. In Cases 1 and 2 the all-equity rate, ρ_0^{PCI}, is equal to the all-equity rate for the firm as a whole (ρ_0). In Cases 3 and 4, ρ_0^{PCI} is the rate of interest on the firm's bonds.

Cases 1 and 3 assume the firm is all-equity financed, while Cases 2 and 4 assume the firm fully utilizes its debt capacity, maintaining a target debt ratio equal to (D/V).

It is not possible to further specify the nature of the PCI APV's without further definition of the PCI cash flows. If we assume, for example, an initial investment I_0 at $t = 0$, and a perpetual stream of annual after-tax payments $-C$ beginning at $t = 1$, then the general expression for the PCI APV is given by

$$APV = \frac{-C}{\rho_0^{PCI}} - I_0 + PVTS \qquad (5\text{-}11)$$

where

ρ_0^{PCI} = the all-equity rate for the PCI

$-C$ = the annual operating cost (after corporate tax)

I_0 = the investment at $t = 0$

PVTS = the present value of the tax savings on the debt capacity reduction caused by the PCI (zero for Cases 1 and 3)

Assuming that perpetual debt has been issued by the firm in Cases 2 and 4, the present value of the tax savings on the debt capacity reductions caused by a PCI is equal to $-T_c \cdot D^{PCI}$, where T_c is the corporate tax rate and D^{PCI} is the reduction in debt capacity caused by the PCI. In effect, if the firm was making no other investments other than the PCI, it would be forced to retire D^{PCI} dollars of perpetual debt on acceptance of the PCI. In practice, however, firms will be carrying out other investment and financing programs along with the pollution-control requirements. Rather than retiring D^{PCI} dollars of debt, the new financing mix would simply be changed to issue more equity and less debt than otherwise. The effect, however, is the same.

Calculation of the PVTS for the PCI's is complicated in Cases 2 and 4 by the dependence of the reduction in debt capacity on the (negative) value of the PCI, which in turn depends on the present value of the tax savings. This can be more clearly seen from the following steps.

From Equation (5-4) the APV^{PCI} is given by

$$APV^{PCI} = NPV(\rho_0) + PVTS$$

Assuming that all debt is perpetual, and that the firm plans to maintain the target debt ratio equal to (D/V), the adoption of the PCI would require that the debt level be reduced by D^{PCI} dollars, where

$$D^{PCI} = \left(\frac{D}{V}\right) \left| APV^{PCI} \right| \qquad (5\text{-}12)$$

The present value of the tax savings associated with this reduction is given by

$$PVTS = T_c \left(\frac{D}{V}\right) (APV^{PCI}) \qquad (5\text{-}13)$$

Substituting from Equation (5-13) into Equation (5-4) we have

$$APV^{PCI} = NPV(\rho_0) + T_c \cdot \frac{D}{V} \cdot APV^{PCI}$$

or, rearranging terms,

$$APV^{PCI} = \frac{1}{\left[1 - T_c \cdot \frac{D}{V}\right]} \cdot NPV(\rho_0) \qquad (5\text{-}14)$$

Thus, reduction in market value of the firm will increase with increasing debt ratios. Specific forms of Equation (5-14) for the four cases are given in Table 5-2.

(c) The Offset Rate of Return

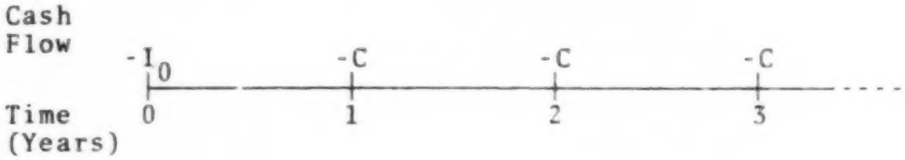
How much would the rate of return on the firm's other assets have to increase to offset the negative impact of the PCI? The effect would be offset when the increased present value resulting from the increase in return was just equal to the (absolute) value of the PCI APV. That is,

$$\Delta APV^{FIRM} = \left| APV^{PCI} \right| \qquad (5\text{-}15)$$

To further specialize Equation (5-15), further assumptions regarding the firm's assets are required.

TABLE 5-2

ADJUSTED PRESENT VALUE FOR EXAMPLE POLLUTION-CONTROL INVESTMENT


Cash Flow
$-I_0$
$-C$
$-C$
$-C$
Time (Years)
0
1
2
3

Case	Adjusted Present Value (APV)
1 Eq. (5-14a)	$\frac{-C}{\rho_0} - I_0$
2 Eq. (5-14b)	$\frac{1}{\left(1 - T_c \cdot \frac{D}{V}\right)} \left[\frac{-C}{\rho_0} - I_0\right]$
3 Eq. (5-14c)	$\frac{-C}{i} - I_0$
4 Eq. (5-14d)	$\frac{1}{\left(1 - T_c \cdot \frac{D}{V}\right)} \left[\frac{-C}{i} - I_0\right]$

Assume, for example, the market value of the firm's assets is V_B (prior to adoption of the PCI). The assets generate a perpetual after-tax return of r percent per year. Also assume the return can be increased by Δr percent per year; hence, the increased annual cash flow is $\Delta r\ V_B$ dollars per year. Assume further the firm will adjust its capital structure to maintain its target debt ratio (and hence the appropriate discount rate rate of ρ^*).

$$\frac{\Delta r \cdot V_B}{\rho^*} = \left|APV^{PCI}\right|$$

Therefore,

$$\Delta r = \frac{\left|APV^{PCI}\right|}{V_B} \cdot \rho^*$$

or

$$\Delta r = (FII)\ \rho^* \qquad (5\text{-}16)$$

where

ρ^* = the (textbook) weighted average cost of capital

Thus, given the above assumptions, the necessary increase in the return on the firm's other assets is equal to the financial impact index (FII) times the weighted average cost of capital. (Note that for an all-equity financed firm, $\rho^* = \rho_0$ and hence $\Delta r = (FII)\ \rho_0$.)

(d) Example

Consider three firms (A, B, and C) with market values of assets prior to adoption of PCI's (V_B) of 1000, 500, and 100, respectively. Each firm generates a perpetual return on the market value of its assets of 10 percent per year. The all-equity rate for average-risk projects is 10 percent, and the firms can borrow at a 6-percent rate. The corporate tax rate (T_c is 50 percent. The firms are assumed to be identical except for the size of assets.

Each firm is required to install a polution-control device at $t = 0$. The required investments are 100, 75, and 40 for the three firms respectively. Annual operating costs are 10, 5, and 1.

For each firm we will consider the implications of the four cases defined in Table 5-1. In Cases 1 and 3 we assume the PCI's to be of average risk. Therefore, $\rho_0^{PCI} = 0.10$. In Cases 2 and 4 we assume the PCI's to be of minimal risk (same risk as the firm's bonds). Therefore, $\rho_0^{PCI} = 0.06$.

In Cases 1 and 3 it is assumed that the three firms are all-equity financed (and plan to remain so in the future). Thus, PVTS = 0 and $\rho^* = \rho_0 = 0.10$. In Cases 2 and 4 it is assumed that the three firms have fully utilized all of the debt capacity of existing assets. Each firm has a market value debt ratio (D/V) equal to 30 percent which will be maintained after the PCI. In order to achieve this, each firm will be required to retire debt equal to 30 percent of the (negative) market value of the PCI through new equity issues (see Equation (5-12)). This is in addition to the equity issued to finance the PCI initial investment (I_0 dollars). Each firm has a weighted average cost of capital of 8.5 percent ($\rho^* = 0.08$).

The results for the four cases and three firms are summarized in Table 5-3. Row 7 of Table 5-3 gives the financial impact index values; row 9 gives the offset rates of return.

While the numbers in the example were selected for illustration only, the results clearly show the relationship between the effect of pollution investments and the size of firm. It seems fairly clear a small firm such as C would be forced out of business, while the largest firm A would have the best chance of successfully absorbing the PCI.

Note that the offset rates of return do not depend on the capital structures of the firms (see row 9, Table 5-3). Even though the initial impact of the PCI is more severe for the debt firms (see row 7) due to the loss of valuable debt capacity, it is exactly offset by the new debt capacity generated by the increased return (Δr) on the firm's other assets. This, of course, assumes the offset returns can be achieved. If not, the impact remains greater for the firms using debt.

TABLE 5-3

EXAMPLE: THE IMPACT OF POLLUTION-CONTROL INVESTMENTS
ON FIRMS OF DIFFERENT SIZE*

	Average-Risk Investment (Cases 1 and 2)						Low-Risk Investment (Cases 3 and 4)					
	Case 1 All Equity			Case 2 30% Debt Ratio			Case 3 All Equity			Case 4 30% Debt Ratio		
	Firm A	Firm B	Firm C	Firm A	Firm B	Firm C	Firm A	Firm B	Firm C	Firm A	Firm B	Firm C
Market Value of Firm (V_B)	1000	500	100	1000	500	100	1000	500	100	1000	500	100
PC Investment (T = 0)	-100	-75	-40	-100	-75	-40	-100	-75	-40	-100	-75	-40
PC Annual Cost (t = 1,∞)	-10	-5	-1	-10	-5	-1	-10	-5	-1	-10	-5	-1
Base Case NPV	-200	-125	-50	-200	-125	-50	-266.7	-158.3	-56.7	-266.7	-158.3	-56.7
PVTS	0	0	0	-35.3	-22.0	-8.8	0	0	0	-47.1	-27.9	-16.7
APV = NPV + PVTS	-200	-125	-50	-235.3	-147.	-58.8	-166.7	-158.3	-56.7	-313.7	-186.2	-66.7
Impact Index	.200	.250	.500	.235	.294	.588	.267	.317	.567	.314	.372	.667
Offset ΔR (%)	2.0	2.5	5.0	2.0	2.5	5.0	2.7	3.2	5.7	2.7	3.2	5.7
Offset Return (%)	12.0	12.5	15.0	12.0	12.5	15.0	12.7	13.2	15.7	12.7	13.2	15.7

* The numbers are chosen for illustrative purposes only.

The relationships between the impact index and size are displayed in Figure 5-1. As shown in Figure 5-1, the relationship is very dependent on both the risk of the PCI and the firms' capital structures.

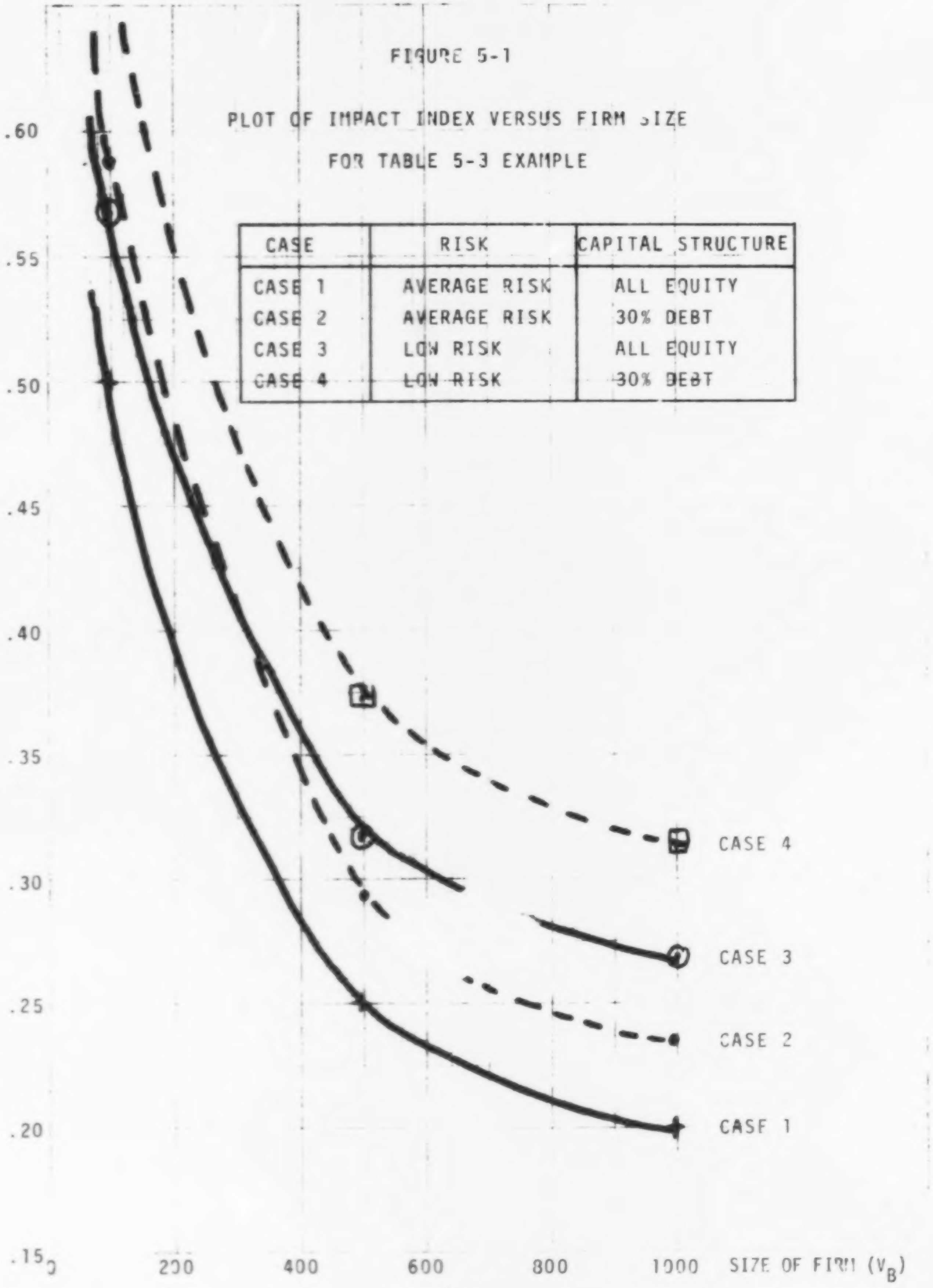


FIGURE 5-1

PLOT OF IMPACT INDEX VERSUS FIRM SIZE

FOR TABLE 5-3 EXAMPLE

CASE	RISK	CAPITAL STRUCTURE
CASE 1	AVERAGE RISK	ALL EQUITY
CASE 2	AVERAGE RISK	30% DEBT
CASE 3	LOW RISK	ALL EQUITY
CASE 4	LOW RISK	30% DEBT

FOOTNOTES FOR PART 5

1. The adjusted-present-value approach to capital budgeting was developed by Professor Stewart C. Myers and is described in detail in [21].

2. See Hamada [6], p. 20, Equations 15, 16, and 17. The ratio of Equations (15) and (16) gives (in the notation of this study)

$$\frac{B}{B_0} = \frac{E_0}{E}$$

where E_0 is the total value of the unlevered equity, E the levered equity. By Equation (17)

$$E_0 = E + (1 - T_c)D$$

or

$$\frac{E_0}{E} = 1 + (1 - T_c) \left(\frac{D}{E}\right)$$

Thus, combining these two results,

$$B = B_0\left[1 + (1 - T_c) \left(\frac{D}{E}\right)\right]$$

as given in Equation (5-9).

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